Math 421 / Homework 8.4

 # 2 For each of the following sets, sketch E^o, $\bar{E}$, and ∂E.

(a) $E = \{(x, y) :\ x^2 + 4y^2 \le 1\}$

(b) $E = \{(x, y) :\ x^2 - 2x + y^2 = 0\} \cup \{(x, 0) :\ x \in [2, 3]\}$

(c) $E = \{(x, y) :\ y \ge x^2,\ 0 \le y < 1\}$

(d) $E = \{(x, y) :\ x^2 - y^2 < 1,\ -1 < y < 1\}$

 # 3 Suppose that $A \subseteq B \subseteq \mathbf{R}^n$. Prove that

$$\bar{A} \subseteq \bar{B}, \quad A^o \subseteq B^o.$$

 # 4 Let E be a subset of $\mathbf{R}^n$.

(a) Prove that every subset $A \subseteq E$ contains a set B which is the largest subset of A that is relatively open in E.

(b) Prove that every subset $A \subseteq E$ is contained in a set B which is the smallest subset of E containing A that is relatively closed in E.

 # 7 Suppose that $E \subset \mathbf{R}^n$ is connected and that $E \subseteq A \subseteq \bar{E}$. Prove that A is connected.

 # 9 Show that Theorem 8.37 is best possible in the following sense.

(a) There exist sets A, B in $\mathbf{R}$ such that $(A \cup B)^o \ne A^o \cup B^o$.

(b) There exist sets A, B in $\mathbf{R}$ such that $\overline{A \cap B} \ne \bar{A} \cap \bar{B}$.

(c) There exist sets A, B in $\mathbf{R}$ such that $\partial(A \cup B) \ne \partial A \cup \partial B$ and $\partial(A \cap B) \ne \partial A \cup \partial B$.