Math 421 / Homework 8.2

 # 2(a) Find an equation of the hyperplane through the points $(1,0,0,0), (2,1,0,0), (0,1,1,0)$ and $(0,4,0,1)$.

 # 3 Find two lines in $\mathbf{R}^3$ which are not parallel but do not intersect.

 # 5 Suppose that $\mathbf{T} \in \mathcal{L}(\mathbf{R}^n;\mathbf{R}^m)$ for some $n,m \in \mathbf{N}$.

(a) If $\mathbf{T}(1,1) = (3,\pi,0)$ and $\mathbf{T}(0,1) = (4,0,1)$, find the matrix representative of $\mathbf{T}$.

(b) If $\mathbf{T}(1,1,0) = (e,\pi), \mathbf{T}(0,-1,1) = (1,0)$, and $\mathbf{T}(1,1,-1) = (1,2)$, find the matrix representative of $\mathbf{T}$.

(c) If $\mathbf{T}(0,1,1,0) = (3,5), \mathbf{T}(0,1,-1,0) = (5,3)$ and $\mathbf{T}(0,0,0,-1) = (\pi,3)$, find the matrix representative of $\mathbf{T}$.

 # 6 Suppose that $\mathbf{a},\mathbf{b},\mathbf{c} \in \mathbf{R}^3$ are three points which do not lie on the same straight line and that Π is the plane which contains the points $\mathbf{a},\mathbf{b},\mathbf{c}$. Prove that an equation of Π is given by

$$\det\begin{bmatrix} x-a_1 & y-a_2 & z-a_3 \\ b_1-a_1 & b_2-a_2 & b_3-a_3 \\ c_1-a_1 & c_2-a_2 & c_3-a_3 \end{bmatrix} = 0.$$

 # 10(a) For $\mathbf{f}(x) = (x^2, \sin x)$, find the matrix representative of a linear transformation $\mathbf{T} \in \mathcal{L}(\mathbf{R};\mathbf{R}^2)$ which satisfies

$$\lim_{h\to 0} \frac{\|\mathbf{f}(x+h) - \mathbf{f}(x) - \mathbf{T}(h)\|}{h} = 0.$$

 # 11 Fix $\mathbf{T} \in \mathcal{L}(\mathbf{R}^n;\mathbf{R}^m)$. Set

$$M_1 := \sup_{\|\mathbf{x}\|=1} \|\mathbf{T}(\mathbf{x})\| \quad \text{and}$$

$$M_2 := \inf\{C > 0 \;:\; \|\mathbf{T}(\mathbf{x})\| \le C\|\mathbf{x}\| \;\; \forall \mathbf{x} \in \mathbf{R}^n\}.$$

(a) Prove that $M_1 \le \|\mathbf{T}\|$.

(b) Using the linear property of $\mathbf{T}$, prove that if $\mathbf{x} \neq 0$, then

$$\frac{\|\mathbf{T}(\mathbf{x})\|}{\|\mathbf{x}\|} \le M_1.$$

(c) Prove that $M_1 = M_2 = \|\mathbf{T}\|$.