Math 421 / Homework 11.5

 # 1 (a) Write out an expression in powers of $(x+1)$ and $(y-1)$ for $f(x,y) = x^2 + xy + y^2$.

(b) Write Taylor's Formula for $f(x,y) = \sqrt{x} + \sqrt{y}$ at $\mathbf{a} = (1,4)$ with degree $p = 3$.

(c) Write Taylor's Formula for $f(x,y) = e^{xy}$ at $\mathbf{a} = (0,0)$ with degree $p = 4$.

 # 7 Suppose that $0 < r < 1$ and that $f \colon B_1(\mathbf{0}) \to \mathbf{R}$ is continuously differentiable. If there is an $\alpha > 0$ such that $|f(\mathbf{x})| \le \|\mathbf{x}\|^\alpha$ for all $\mathbf{x} \in B_r(\mathbf{0})$, prove that there exists an $M > 0$ such that $|f(\mathbf{x})| \le M\|\mathbf{x}\|$ for all $\mathbf{x} \in B_r(\mathbf{0})$.

 # 8 Suppose that V is open in $\mathbf{R}^n$, that $f \colon V \to \mathbf{R}$ is $\mathcal{C}^2$ on V, and that $f_{x_j}(\mathbf{a}) = 0$ for some $\mathbf{a} \in H$ and all $j = 1, 2, \dots, n$. Prove that if H is a compact convex subset of V, then there exists a constant $M > 0$ such that

$$|f(\mathbf{x}) - f(\mathbf{a})| \le M\|\mathbf{x} - \mathbf{a}\|^2 \quad \forall\, \mathbf{x} \in H.$$