Math 421 / Homework 11.4

 # 1 Let $F\colon \mathbf{R}^3 \to \mathbf{R}$ and $f, g, h\colon \mathbf{R}^2 \to \mathbf{R}$ be C^2 functions. If $w = F(x, y, z)$, where $x = f(p, q)$, $y = g(p, q)$ and $z = h(p, q)$, find formulas for w_p, w_q, and w_{pp}.

 # 4 Let $f, g\colon \mathbf{R} \to \mathbf{R}$ be twice differentiable. Prove that $u(x, y) := f(xy)$ satisfies

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0,$$

and $v(x, y) := f(x - y) + g(x + y)$ satisfies the *wave equation*; that is,

$$\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} = 0.$$

 # 7 Let

$$u(x, t) = \frac{e^{-x^2/4t}}{\sqrt{4\pi t}}, \quad t > 0, \ x \in \mathbf{R}.$$

(a) Prove that u satisfies the *heat equation*; that is, $u_{xx} - u_t = 0$ for all $t > 0$ and $x \in \mathbf{R}$.

(b) If $a > 0$, prove that $u(x, t) \to 0$ as $t \to 0^+$ uniformly for $x \in [a, \infty)$.