

Math 421 / Homework 11.1

2 For each of the following functions, compute f_x and determine where it is continuous.

(a)

$$f(x, y) = \begin{cases} \frac{x^4 + y^4}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0). \end{cases}$$

(b)

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{\sqrt[3]{x^2 + y^2}} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0). \end{cases}$$

4 Suppose that $H = [a, b] \times [c, d]$ is a rectangle, that $f: H \rightarrow \mathbf{R}$ is continuous, and that $g: [a, b] \rightarrow \mathbf{R}$ is integrable. Prove that

$$F(y) = \int_a^b g(x)f(x, y) dx$$

is uniformly continuous on $[c, d]$.

5 Evaluate the following expressions.

(a)

$$\lim_{y \rightarrow 0} \int_0^1 e^{x^3 y^2 + x} dx$$

(b)

$$\frac{d}{dy} \int_0^1 \sin(e^x y - y^3 + \pi - e^x) dx \quad y = 1$$

(c)

$$\frac{\partial}{\partial x} \int_1^3 \sqrt{x^3 + y^3 + z^3 - 2} dz \quad \text{at } (x, y) = (1, 1)$$