§ 3.4

3. (a) Proof: $\det\begin{pmatrix}\vec{x}_1 & \vec{x}_2\end{pmatrix} = \det\begin{pmatrix}2 & 4\\ 1 & 3\end{pmatrix}$

$= 2\cdot 3 - 4\cdot 1 = 2 \neq 0$

Thus, $\vec{x}_1$ & $\vec{x}_2$ form a basis for $\mathbb{R}^2$ #

(b) Sol. $\vec{x}_1, \vec{x}_2$ is a basis of $\mathbb{R}^2$

$\Rightarrow \{\vec{x}_1, \vec{x}_2\}$ is a spanning set of $\mathbb{R}^2$

$\xRightarrow[\text{Thm 3.4.1}]{3>2}$ $\vec{x}_1, \vec{x}_2$ & $\vec{x}_3$ are <u>LD</u>. #

(c) Sol: $\vec{x}_1, \vec{x}_2, \vec{x}_3 \in \mathbb{R}^2 \Rightarrow \operatorname{span}\{\vec{x}_1, \vec{x}_2, \vec{x}_3\} \subseteq \mathbb{R}^2$

$\Rightarrow \dim \operatorname{Span}\{\vec{x}_1, \vec{x}_2, \vec{x}_3\} \leq \dim \mathbb{R}^2 = 2$

$\vec{x}_1$ & $\vec{x}_2$ is a basis of $\mathbb{R}^2 \Rightarrow \mathbb{R}^2 \subseteq \operatorname{span}\{\vec{x}_1, \vec{x}_2, \vec{x}_3\}$

$\Rightarrow \dim \operatorname{Span}\{\vec{x}_1, \vec{x}_2, \vec{x}_3\} \geq 2$

Thus. $\dim \operatorname{Span}\{\vec{x}_1, \vec{x}_2, \vec{x}_3\} = 2$

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4 Sol. Note that $\vec{x}_2 = -\vec{x}_1$ ...... ①

$\vec{x}_3 = -2\vec{x}_1$ ...... ②

Thus, $\text{Span}\{\vec{x}_1, \vec{x}_2, \vec{x}_3\} = \{\alpha\vec{x}_1 + \beta\vec{x}_2 + \gamma\vec{x}_3 \mid \alpha, \beta, \gamma \in \mathbb{R}\}$

$\overset{①\&②}{=} \{(\alpha - \beta - 2\gamma)\vec{x}_1 \mid \alpha, \beta, \gamma \in \mathbb{R}\}$

$= \{\lambda\vec{x}_1 \mid \lambda \in \mathbb{R}\}$

$= \text{Span}\{\vec{x}_1\}$

As a result, $\dim(\text{Span}\{\vec{x}_1, \vec{x}_2, \vec{x}_3\}) = \dim(\text{Span}\{\vec{x}_1\}) = 1$

#.

13. Sol. According to the double angle formula of $\cos x$

$$\cos 2x = 2\cos^2 x - 1$$

Thus, $\cos 2x$ is a linear combination of $\cos^2 x$ & 1.

Moreover. $W[\cos^2 x, 1](x) = \begin{vmatrix} \cos^2 x & 1 \\ -\sin 2x & 0 \end{vmatrix}$

$$= \sin 2x$$

Note that $W[\cos^2 x, 1](\frac{\pi}{4}) = 1 \neq 0$

We conclude that $\cos^2 x$ & 1 are linearly independ

As a result, $\dim \text{Span}\{1, \cos^2 x, \cos 2x\} = 2$.

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