You will be given 120 minutes to complete this exam. The exam consists of 27 questions on 7 pages. Some formulae are listed on page 8. Request a new copy of the exam if any of the problems are missing or hard to read.
You must show your work when directed to do so or credit will not be given. If you use a graph from your calculator, sketch it on your exam paper, along with enough markings on the axes to indicate your viewing rectangle.
Questions 1 – 20 are multiple choice (5 pts each). Circle the correct answer. There is only one correct answer for each problem.

1. What is the domain of \( f(x) = \frac{2}{\sqrt{x + 10}} \) ?
   (A) \( x < 10 \)  (B) \( x \geq -10 \)  (C) \( x > -10 \)  (D) All real numbers

2. How many x-intercepts does \( g(x) = x^2 + 2x + 10 \) have?
   (A) One  (B) None  (C) Two

3. Which of the following equations represents the line with undefined slope?
   (A) \( y = 4 \)  (B) \( y = 2x - 5 \)  (C) \( y = x \)  (D) \( x = -3 \)  (E) None of these

4. Which of the following points represents the vertex of \( y = x^2 - 4 \) ?
   (A) \( (0, -4) \)  (B) \( (4, 0) \)  (C) \( (0, 4) \)  (D) \( (-4, 0) \)  (E) None of these

5. Which of the following equations is equivalent to \( \ln x = 3 \) ? (Assume that \( x > 0 \))
   (A) \( x = 3^e \)  (B) \( x = e^3 \)  (C) \( x^3 = e \)  (D) None of these

6. Which of the following is equivalent to \( 2 \log x - 5 \log y + \log z \) ? (Assume that \( x, y, z > 0 \))
   (A) \( \log(2x - 5y + z) \)  (B) \( \log\left(\frac{2xz}{5y}\right) \)  (C) \( \log\left(\frac{x^2z}{y^5}\right) \)  (D) None of these

7. What is the monthly payment (to the nearest dollar) on a $20,000 car loan for 5 years at 7% compounded monthly?
   (A) $245  (B) $350  (C) $403  (D) $396  (E) None of these
8. Suppose you invested $1,000 into each of the following accounts: account X which pays 4.5% compounded monthly, and account Y which pays 4.8% compounded annually. Which one of the two accounts will have more money in it at the end of 5 years (no money withdrawn)?
(A) Account X  (B) Account Y

9. Consider the system of linear equations: \[2x - 3y = 4\]
\[4x - 6y = 7\]. How many solutions does this system have?
(A) Infinitely many  (B) One  (C) Two  (D) None

10. Which one of the following matrices is not in reduced form?
(A) \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 0 & 1 \\
\end{bmatrix}
\]  (B) \[
\begin{bmatrix}
1 & 0 & 7 \\
0 & 1 & 2 \\
0 & 0 & 0 \\
\end{bmatrix}
\]  (C) \[
\begin{bmatrix}
1 & 4 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]  (D) None of these

11. A single fair die is rolled once. What is the probability of rolling a 2 or a 6?
(A) \(\frac{1}{3}\)  (B) \(\frac{1}{2}\)  (C) \(\frac{1}{36}\)  (D) None of these

12. What is the probability that a 5-card hand from a standard 52-card deck will have three diamonds?
(A) 0.0054  (B) 0.017  (C) 0.00011  (D) None of these

13. Given that the probability of event E is 0.07, what is the probability of the complement of E?
(A) -0.07  (B) 0.93  (C) -0.93  (D) None of these

14. What are the odds in favor of event M if \(P(M) = 0.3\)?
15. Suppose events \( F, G, H \) are independent. Find \( P(F \cap G \cap H) \) if \( P(F) = 0.5, P(G) = 0.4, \) and \( P(H) = 0.1 \).

(A) 0.2  (B) 0.04  (C) 0.02  (D) None of these

16. A spinner is numbered from 1 through 8 with each number as likely to occur as any other. Compute the probability that in a single spin the dial will stop at number less than 3 or greater than 6.

(A) 0.5  (B) 0.25  (C) \( \frac{1}{8} \)  (D) None of these

17. Consider the sample space \( S = \{X, Y, Z\} \). Determine if the following probability assignment is acceptable: \( P(X) = 0.2, P(Y) = 3, P(Z) = 0.8 \).

(A) Acceptable  (B) Not acceptable

18. Find the mean for the data set: 2, 11, 35, 2, 9, 7, 8.

(A) 8  (B) 10.57  (C) 9.3  (D) None of these

19. Find the median for the data set: 2, 8, 2, 5, 11, 8, 13.

(A) 5  (B) 8  (C) 7  (D) None of these

20. Find the mode for the data set: 4, 2, 3, 2, 4, 3, 3, 7, 11, 12, 7.

(A) 3  (B) 2  (C) 3.5  (D) None of these
21. (15 pts) Maximize and minimize \( P = 3x_1 + 2x_2 \)

\[
\begin{align*}
5x_1 + 2x_2 & \leq 40 \\
x_1 + 3x_2 & \leq 21 \\
x_1, x_2 & \geq 0
\end{align*}
\]

Subject to

Show work.

\[
\begin{align*}
5x_1 + 2x_2 = 40 & \\
-x_1 + 3x_2 = -21 & \\
x_1, x_2 & \geq 0
\end{align*}
\]

\[
\begin{array}{c|c|c}
\text{C.P.} & \text{P} = 3x_1 + 2x_2 & \\
\hline
(0,0) & P = 0 & \\
(0,7) & P = 14 & \\
(8,0) & P = 24 & \\
(6,5) & P = 18 + 10 = 28 &
\end{array}
\]

The maximum \( P = 28 \) at C.P. \((6,5)\). The minimum \( P = 0 \) at C.P. \((0,0)\).

22. (15 pts) Solve the following equation: \( \log_7 x + \log_7 (x - 2) = \log_7 24 \). Show work.

\[
\log_7 x(x-2) = \log_7 24
\]

\[
x(x-2) = 24
\]

\[
x^2 - 2x - 24 = 0
\]

\[
(x - 6)(x + 4) = 0
\]

\[
x - 6 = 0 \quad x + 4 = 0
\]

\[
x = 6 \quad x = -4
\]

\[
\text{Check:}
\]

\[
x = 6: \log_7 6 + \log_7 4 = \log_7 24
\]

\[
x = -4: \log_7 (-4) + \log_7 (-6) \neq \log_7 24
\]

\[
\text{NO, we cannot use}
\]

\[
\log_7 6 \text{ or } \log_7 (-4)
\]

\[
\text{Answer:} \boxed{6}
\]

\[
\text{Answer:} \boxed{6}
\]
23. (15 pts) Solve the following system using Gauss–Jordan Elimination. Be sure to indicate each one of your steps.

\[
\begin{align*}
x_1 - 3x_2 - 3x_3 &= 13 \\
x_2 - 4x_3 &= 5 \\
x_1 + x_3 &= -4 \\
\end{align*}
\]

Answer

\[
\begin{bmatrix}
1 & -3 & -3 & | & 13 \\
0 & 1 & -4 & | & 5 \\
1 & 0 & 1 & | & -4 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -3 & -3 & | & 13 \\
0 & 1 & -4 & | & 5 \\
0 & 3 & 4 & | & -17 \\
\end{bmatrix} \Rightarrow 
\begin{bmatrix}
1 & 0 & -15 & | & 28 \\
0 & 1 & -4 & | & 5 \\
0 & 0 & 1 & | & -32 \\
\end{bmatrix} \Rightarrow 
\begin{bmatrix}
1 & 0 & -15 & | & 28 \\
0 & 1 & -4 & | & 5 \\
0 & 0 & 1 & | & -32 \\
\end{bmatrix}
\]

Answer \((-2, -3, z_2)\)

24. (10 pts) A person makes monthly deposits of $500, which earn 5% compounded monthly. At the end of one year, how much interest has been earned? Show work.

\[
FV = 500 \left[ \frac{(1 + \frac{.05}{12})^{12} - 1}{\frac{.05}{12}} \right] = 61,139.43
\]

\[
A = P + I
\]

\[
A = 61,139.43
\]

\[
I = 61,139.43 - 6,000 = 53,139.43
\]
25. (15 pts) By testing a large number of individuals, it has been determined that 80% of the population have normal hearts, 15% have some minor heart problems, and 5% have severe heart problems. Ninety-two percent of the persons with normal hearts, 30% of those with minor problems, and 5% of those with severe problems will pass a cardiogram test. What is the probability that a person who passes the cardiogram test has a normal heart? Show work.

\[
P(\text{N|T}) = \frac{P(\text{N|T})}{P(T)} = \frac{(0.8 \cdot 0.92)}{(0.8 \cdot 0.92) + (0.15 \cdot 0.8) + (0.05 \cdot 0.05)}
\]

\[\text{Answer} = 0.9394\]

26. (15 pts) Find the mean and standard deviation for the following sample data set. Show work.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5 – 12.5</td>
<td>3</td>
</tr>
<tr>
<td>12.5 – 15.5</td>
<td>2</td>
</tr>
<tr>
<td>15.5 – 18.5</td>
<td>7</td>
</tr>
<tr>
<td>18.5 – 21.5</td>
<td>4</td>
</tr>
</tbody>
</table>
27. (15 pts) Sketch the graph corresponding to each of the following descriptions.

(a) \( y = b^{-x}, \ b > 1 \)

(b) A quadratic function with \( a < 0 \) and one \( x \) - intercept.

(c) An even - degree polynomial with 3 turning points.
Interesting Formulae:
\[ A = P(1 + rt) \]
\[ A = P(1 + i)^n \]
\[ FV = PMT \frac{(1+i)^n - 1}{i} \]
\[ PV = PMT \frac{1 - (1+i)^{-n}}{i} \]
\[ \begin{align*}
P_{n,r} &= \frac{n!}{(n-r)!} \\
C_{n,r} &= \frac{n!}{r!(n-r)!} \\
P(E) &= \frac{n(E)}{n(S)} \\
P(E) &= \frac{a}{a+b} \\
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
P(A \mid B) &= \frac{P(A \cap B)}{P(B)} \\
P(A \cap B) &= P(A)P(B \mid A) = P(B)P(A \mid B) \\
P(E') &= 1 - P(E) \\
\bar{x} &= \frac{x_1 + x_2 + \ldots + x_n}{n} \\
\bar{X} &= \frac{\bar{x}_1 f_1 + \bar{x}_2 f_2 + \ldots \bar{x}_n f_n}{n} \\
s^2 &= \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n-1} \\
s^2 &= \frac{(x_1 - \bar{x})^2 f_1 + (x_2 - \bar{x})^2 f_2 + \ldots + (x_k - \bar{x})^2 f_k}{n-1} \\
s &= \sqrt{s^2} \]