4.4 Applications of Logarithmic Functions

In this section you will learn to:
- use logarithms to solve geology problems
- use logarithms to solve charging battery problems
- use logarithms to solve population growth problems

| Richter Scale | If $R$ is the intensity of an earthquake, $A$ is the amplitude (measured in micrometers), and $P$ is the period of time (the time of one oscillation of the Earth's surface, measured in seconds), then 

$$R = \log \frac{A}{P}$$

| Charging Batteries | If $M$ is the theoretical maximum charge that a battery can hold and $k$ is a positive constant that depends on the battery and the charger, the length of time $t$ (in minutes) required to charge the battery to a given level $C$ is given by

$$t = -\frac{1}{k} \ln \left(1 - \frac{C}{M}\right)$$

| Population Doubling Time | If $r$ is the annual growth rate and $t$ is the time (in years) required for a population to double, then

$$t = \frac{\ln 2}{r}$$

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**Example 1:** Find the intensity of an earthquake with amplitude of 4000 micrometers and a period of 0.07 second.

$$R = \log \left(\frac{4000}{0.07}\right) = 4.756961951$$

nearest tenth place $\Rightarrow 1$ dec.

measure 4.8 on Richter Scale

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**Example 2:** An earthquake has a period of $\frac{1}{4}$ second and an amplitude of 6 cm. Find its measure on the Richter scale. (Hint: 1 cm = 10,000 micrometers.)

$$R = \log \left(\frac{60,000}{125}\right) = 5.380211242 \Rightarrow 1$$.dec. place

measure 5.4 on Richter Scale

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**Example 3:** How long will it take to bring a fully discharged battery to 80\% of full charge? Assume that $k = 0.025$ and that time is measured in minutes.

$$t = -\frac{1}{0.025} \ln \left(1 - \frac{80\%}{100}\right) = 14.3775165 = 14.4$$.minutes

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**Example 4:** The population of the Earth is growing at the approximate rate of 1.7\% per year. If this rate continues, how long will it take the population to double?

$$r = 1.7\% \Rightarrow \frac{1.7}{100} \Rightarrow 0.017$$

$$t = \frac{\ln 2}{r} = \frac{\ln 2}{0.017} = 40.77336356$$

It will take about 41 years to bring a fully discharged battery to 80\% of full charge.

We will talk about 41 yrs. for the pop. to double.