A study of Core-Plus students attending Michigan State University

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Abstract

One measure of the effectiveness of a high school mathematics program is the success students have in subsequent university mathematics courses. As part of a large-scale study of Michigan students, we analyzed the records of students arriving at Michigan State University from four high schools which adopted the Core-Plus Mathematics program. Those students placed into, and enrolled in, increasingly lower level courses as the implementation progressed; the downward trend is statistically very robust ($p < .0005$). The grades these students earned in their university mathematics courses were also below average ($p < .01$). ACT scores suggested the existence but not the severity of these trends.

Over the past two decades there has been a growing awareness of the inadequacy of the mathematical skills of American high school graduates. That was the assessment of the 1983 report *A Nation at Risk* [9]. Many subsequent studies point to the same conclusion. The most recent National Assessment of Educational Progress (NAEP) Report [2] concluded that only 17 percent of US twelfth graders were proficient at mathematics[1]. International comparisons also indicate a relatively low level of mathematics achievement by US high school students. The Third International Mathematics and Science Study (TIMSS) assessed the 'Mathematics Literacy' of end-of -secondary students in 22 countries and found that US students statistically outperformed only two countries, Cyprus and South Africa [13]. Related studies suggest that the mathematics courses taken by American high school students are often at a lower level than those taken by their international peers, and that US high schools are offering a wide assortment of courses which lack the focus and coherence found in many foreign curricula [14]. This situation has been of particular concern on college and university campuses, where large numbers of students require remedial courses to bring their mathematical knowledge and skills up to what is required for college-level mathematics and science courses.

One effort to improve school mathematics began in 1989 with the *Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics

[1]This statistic uses the notion of 'proficient', identified by the National Assessment Governing Board as the level that all students should reach.

[10]. The National Science Foundation (NSF) subsequently funded the development and implementation of thirteen elementary, middle, and high school mathematics curricula based on the NCTM *Standards*. Many of those programs have been controversial. In particular, there has been concern that the NSF-funded curricula moved from pilot testing to large-scale implementation without sufficient independent evaluations of their efficacy in preparing students for college mathematics and science courses.

College-bound students are not an elite group. Approximately 70% of the graduates of American high schools enroll at a two- or four-year college within two years of graduating [5]. For that reason, the preparation of students for college courses is an obviously important goal of any high school mathematics curriculum. It is surprising, then, that there has been no systematic effort to measure the success of curricula (new and old) by examining how well those curricula prepare students for the next level of their education.

This article reports the results of a study of Michigan State University (MSU) students who graduated from several high schools that adopted one of the NSF-funded curricula: the Core-Plus program (described below). By examining data on student performance in MSU mathematics courses, the study aimed to answer the question: How well does the Core-Plus mathematics curriculum prepare students for mathematics courses at Michigan State University?

This study question is a deliberately narrow and focused instance of a general class of questions: How well does program X prepare students for university mathematics courses? One of our aims is to demonstrate that it is feasible and productive to study these questions. Our work has convinced us that mathematicians can use data available to them from their university and from cooperating high schools, to produce meaningful results.

Statistical studies evaluating academic programs must be carefully designed to deal with the many variables which may affect outcomes. Our study design uses several methods to avoid some standard difficulties. We avoid the need to keep track of which high school mathematics courses were taken by individual students by focusing on high schools which were implementing a single new mathematics program. We avoid most issues of demographic variables by *comparing schools with themselves*. For each high school, we examine the data on the students graduating from that school during years in which the Core-Plus program was implemented, and look for upward or downward trends in various measures of success. We use ACT English scores as an indicator of whether the abilities of the students from each high school, on average, changed over the years of the study. Finally, we use a large control group (consisting of the MSU students coming from 34 high schools) to verify that the level and average grades in MSU courses remained constant over the years of the study. These and other aspects of the study design are discussed further in Sections 2 and 5.

The results of this study, presented in Sections 3 and 4 and summarized in Section 6, indicate that concerns about the Core-Plus program might be well-founded. They also point to the need for careful statistical studies of how well other high school programs, especially new programs, are preparing students for college mathematics.

1 The Core-Plus Program and MSU

One of the NSF-funded curricula is the high school program *Contemporary Mathematics in Context: A Unified Approach* developed by the Core-Plus Mathematics Project based at Western Michigan University. The Core-Plus authors describe the program as follows.

Contemporary Mathematics in Context is a four-year curriculum that replaces the traditional Algebra-Geometry-Advanced Algebra/Trigonometry-Precalculus sequence. Each course features interwoven strands of algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. The first three courses in the series provide a common core of broadly useful mathematics for all students. They were developed to prepare students for success in college, in careers, and in daily life in contemporary society. Course 4 continues the preparation of students for college mathematics.[2]

The Core-Plus curriculum features mathematical modeling, extensive use of graphing calculators, and a minimal amount of algebra until the fourth year[3]. The Core-Plus program was first pilot tested in the 1993-94 school year. It is now used by over 200,000 students in more than 500 schools[4].

Michigan State University is well-suited for a study of this kind. It is a large (43,000 students) public university whose students come from a wide variety of mathematics curricula. Because of its size and popularity, the Core-Plus high schools in this study sent many students to MSU (an average of 21 per year from each school). Moreover, in large part because MSU has a mathematics requirement for graduation, roughly 90% of MSU students take a mathematics course during their freshman year. As a result, MSU records provide a large and varied set of data to analyze.

Literature Review

To our knowledge there are only three studies that examined how Core-Plus students fared in subsequent college mathematics courses. A recent small-scale study by Lewis, Lazarovici, & Smith [7] studied seven Core-Plus students who placed into precalculus or calculus at Michigan State University. For five of those students Calculus 1 was "a major struggle"; several dropped Calculus 1 for another math sequence, and three changed majors from engineering to a major requiring fewer mathematics courses. The sixth student did well in Calculus 1 but did poorly in Calculus 2. The seventh student completed the technical calculus sequence and majored in mechanical engineering. However, that student had taken a traditional precalculus course his senior year which "appeared to be pivotal in [his] success in the calculus" ([7], page 13). To the extent that it is possible to conclude anything from a sample of seven students, this study suggests that the Core-Plus courses may not, by themselves, be adequate preparation for the standard calculus course taught at universities to engineering and science majors.

There are two studies that examine the graduates of two high schools in an affluent Detroit suburb; one of these schools was a pilot site for the Core-Plus program and the other did not use Core-Plus material. Information from a sample of these students was collected and analyzed by mathematicians G. Bachelis and R. J. Milgram [8]. On all four of the measures they examined — ACT Math scores, SAT Math scores, level of college courses taken, and grades in those courses — the students from the Core-Plus school performed less well, by a statistically significant amount, than the students from the non-Core-Plus school. However, those conclusions are based on a non-random sample of student-reported data (volunteered responses from 50% of the graduates at the Core-plus school and 25% of those at the non-Core-Plus school).

[2] This description is taken from the website of the Core-Plus program.

[3] The fourth year of the program was not implemented until the 1998-1999 school year and the Core-Plus Course 3 was later modified to include more algebra.

[4] These figures are cited in an NSF grant proposal by the Core-Plus Mathematics Project.

In a subsequent study, A. Coxford[5] obtained data on the mathematics courses taken and grades received by the students from these same two high schools who subsequently enrolled at the University of Michigan at Ann Arbor. His data are reported by H. Schoen and C. Hirsch, who conclude that the "preliminary evidence suggests that students who experienced the pilot Core-Plus curriculum were at least as well prepared for calculus (AP or college level) as students in a more traditional curriculum" [11]. However, the University of Michigan is a highly selective university[6] and hence this was a population of top students, many of whom had taken AP Calculus in additional to Core-Plus courses. We will comment further on these last two studies and offer a hypothesis about the apparent contradiction between them in Section 4 below.

2 Methodology

This study is a spin-off of a much larger study [6] examining achievement in university mathematics courses by students arriving at Michigan State University in the years 1996-1999. We invited 45 Michigan high schools to participate in that large study. Those that agreed (34 schools) received a list of all students who graduated from their school and came to MSU in that four-year period; the schools returned information on the mathematics courses taken and the grades received by those students in their senior year of high school. We matched that information with similar data on those students' MSU mathematics courses. After examining the data, we sent each high school a confidential report on the performance of their students at MSU. Most schools were very interested in obtaining such a report, and readily agreed to participate. Six agreed but later withdrew, with several saying they had difficulty gathering the required data. Only five schools directly declined to participate; all used the Core-Plus program.

In fact, only one of the six Core-Plus schools we approached agreed to participate in the large study. Nevertheless, the students from those six schools provided a reasonably large group of graduates of schools that implemented this new mathematics curriculum over the years studied, and we thought it would be valuable to know how those Core-Plus students fared at MSU. Thus we decided to initiate a separate study focusing on the students from those six high schools using the MSU data available to us. This article reports on that focused study.

Study Design

We began with a list of all students who graduated from the six Core-Plus high schools in our sample in the years 1996-99 and enrolled at MSU. We obtained, from MSU records, those students' ACT scores, scores on the Mathematics Placement exam (see below), and a list of the mathematics courses they took at MSU and the grades they received. More recent data were not available when we began the study, but we later obtained data for the 1994, 1995, and 2000 students from several of these schools. However, two of the six high schools supplemented the Core-Plus program by incorporating material from other curricula. We separated the data from those two schools and analyzed them separately — see the end of Section 4 and footnote 10.

That left four schools which were implementing a system of offering only Core-Plus mathematics (and possibly AP Calculus for students who had completed the Core-Plus program). These four schools represented a variety of communities: one is in the center of a medium-size city, one in an affluent suburb, and two in a developing rural area. All four implemented the Core-Plus curriculum in such a way that very few of their 1996 graduates had taken Core-Plus

[5]Coxford was one of the Principal Investigators of the original N.S.F. grant awarded to the Core-Plus project.

[6]According to U.S. News and World's *American's Best Colleges 2003* (pages 82-84), 69% of University of Michigan freshmen (verses 26% of MSU freshmen) come from the top 10% of their high school class.

courses, but a significant and increasing number of their 1998 and 1999 students had taken at least three of the four Core-Plus courses and no other mathematics course except possibly AP calculus. The students from these four schools constitute our Core-Plus group (the "CP" group).

The goal of our statistical analysis was to look for trends over time in the college mathematics performance of the graduates of the CP group schools. Did the students who graduated later (the groups who had the most exposure to the Core-Plus program) perform significantly better than those who graduated *from the same high schools* earlier (the groups with less Core-Plus exposure)? To answer that question, we separated the CP group into subgroups according to the year of graduation. For each subgroup, we examined the data from the students' first university mathematics course. We looked for trends over time for several measures of success: level of the course taken, the grade in that course, and success rates for moving on to higher mathematics courses.

Of course, trends over time may be attributable to changes in MSU programs, courses, or policies, or to changes in the population of students enrolling at MSU. To see if that was the case, we did a parallel analysis on data for students from the 34 high schools in our database which did not use the Core-Plus program as their primary mathematics program (the "Control group"). These high schools used a wide variety of mathematics programs and textbooks. We did not attempt to subdivide or classify these, but simply took this large group as representative of the mix of programs being used in Michigan high schools. This control group is essentially a "non-Core-Plus" group[7].

These analyses were feasible and sensible because both groups were reasonably large ($n = 353$ and $n = 2961$, respectively), because we had complete records of the MSU mathematics courses taken by all students in both groups, and because we also had ACT English scores to use as an independent assessment of the abilities of these groups of students. The results are presented in Section 3.

Shortcomings of the methodology

This was not a large-scale randomized study. It involved a total of 353 students from four Core-Plus high schools and a control group of 2961 students. Those data produced statistically reliable conclusions, but it remains possible that the implementation of the Core-Plus program in these particular schools was somehow atypical.

This study involves only students at Michigan State University. While this population is reasonably representative of college students, it does not constitute a random sample of all Core-Plus students. In particular, it does not include students who enrolled at community colleges, who enrolled at elite universities, or who did not go to college. Consequently, this study can say nothing about Core-Plus outcomes for those populations of students.

The lack of information from the Core-Plus high schools caused difficulties for this study. It was impossible to ascertain precisely which students had completed the Core-Plus program and which had also taken non-Core-Plus mathematics courses such as AP calculus. We minimized that problem by restricting attention to schools with significantly increasing numbers of students taking Core-Plus courses and looking for trends over time. Nevertheless, the graduates of those schools still include, even in the last year studied, some students who took non-Core-Plus mathematics courses. A more refined study would distinguish those students.

There are also issues involving timing. The Core-Plus program is usually implemented by starting with Course 1 for ninth grade students and adding the subsequent courses one year at a

[7] However, as part of the mix, it includes a small percentage (3.1%) of students who had taken at least one Core-Plus course. These students were retained in the control group to maintain its representative character.

time. In principle that creates a sudden shift, with one year's graduates having taken no Core-Plus courses, and the following year's graduates having taken three or four Core-Plus courses. In our sample of four high schools, the year of the shift varies from school to school, and the shift occurs a year later for the "advanced" students (those who begin high school mathematics in Grade 8). For two of the schools, there was an intermediate class in which roughly half the students began Core-Plus in Grade 9. Our analysis deals with this in two ways: by analyzing individual schools and by looking for log-linear trends in the composite data.

It is possible that observed trends are reflections of changing demographics in the population of the schools studied or in the ability levels of the students who chose to come to MSU. We looked for such treads by the standard method of examining ACT *English* scores. Those scores (see Tables 5 and 8) indicated, but of course did not prove, that there were no substantial changes in the general academic level of the students in the Core-Plus group over the years studied.

Another timing issue concerns the Core-Plus program itself. The program and the manner of its implementation have been evolving. Core-Plus Course 4 was not implemented until the last year of this study, some schools that began implementing Core-Plus as their only curriculum later incorporated additional material in various ways, and there is now a second edition of the Core-Plus texts. We did not examine any details of the implementations, such as teacher training. These are complicated issues that were impossible to sort out with the data we had.

Finally, there is the problem of causal inference. While this study reports compelling trends in the data from the schools implementing Core-Plus, and while those trends are not present in the Control group, there is no guarantee that the trends are due to the Core-Plus program. All of the results of this paper must be read with that caveat in mind.

Freshman Mathematics Courses at MSU

For this study, we have grouped the freshman-level MSU mathematics courses into five tiers. The following discussion describes those tiers and gives background information about the MSU mathematics program.

Nearly 7000 freshmen enter MSU each fall. It is crucial that these students are placed into mathematics courses that are at a level appropriate for them. That is done through a Mathematics Placement Exam designed by the MSU Mathematics Department. The exam is evaluated each year (and revised when appropriate) using several different standard statistical checks of the correlation between Placement Exam scores and the grades students achieve in their freshman courses. These internal Mathematics Department studies show that the Placement Exam directs students into courses where they can succeed.

Here is a brief description of the five tiers, including the Placement Exam score (out of a maximum score of 28) needed to place into the courses in that tier. These are all one-semester courses.

- Tier 1: Math 132 (technical calculus) or a higher level course. Admission into these courses requires a score of 19 on the Placement Exam, an ACT math score of 28, a SAT math score of 640, or a passing score on the AP calculus exam.

- Tier 2: Five different courses. Specifically, (a) business/biological science calculus, (b) trigonometry, (c) elementary education math, (d) a statistics course, and (e) a recently created "liberal arts math" course. Admission into these courses requires a Placement Exam score of 15 (except the trigonometry course, which requires credit for college algebra).

- Tier 3: Precalculus. A traditional precalculus course designed to prepare students for technical calculus, although about half of the students are in there for other purposes. Admission into this course requires a Placement Exam score of 12.
- Tier 4: A traditional "College Algebra" course for three credits and a five-credit course on finite math and algebra. A Placement Exam score of 10 or higher places students into this tier.
- Tier 5: Intermediate algebra, which is a remedial math course for college students. It is a three-credit course that does not earn credit towards graduation but it does count towards a student's GPA.

As a requirement for graduation, all MSU students must pass one mathematics course beyond the level of college algebra. Many large universities have similar graduation requirements. This requirement can be met either by scoring at least 19 on the (proctored) Placement Exam or by receiving a passing grade in any mathematics or statistics course except for College Algebra and the Tier 5 course[8]. The latter two courses cover material commonly found in high school mathematics curricula.

Nearly all of the students in the courses listed above intend to major in something other than mathematics. Consequently, representatives of the mathematics department periodically consult with other departments about the content of these courses. Feedback from these meetings is used to ensure that these courses are meeting the needs of the other departments.

For this study, students were assigned to the tier of the first mathematics course they took at MSU. Students who took no MSU mathematics course were placed either into the tier of the highest-level course for which they had received transfer or AP credit, or into a separate group labeled "None" if they had no such credit.

3 Results

We analyzed the data from the Core-Plus and the Control groups, looking for trends over time in three measures of performance in university mathematics: the level of the first mathematics courses that students took at MSU, the grades they attained in those courses, and the percent of students who eventually passed a Tier 1 course at MSU. We focused on the students' *first* mathematics courses and grades because those most strongly reflect the level of the students' mathematics preparation in high school. The "eventually passed Tier 1" data is a measure of the rate at which students in these groups are taking the mathematics courses needed for technical majors such as engineering, physical science, and economics. This section presents the results of a statistical analysis of these three measures.

Courses Taken

At the beginning of their freshman year, the students themselves choose their mathematics courses from the options available to them as determined by their Placement Exam scores, ACT or SAT scores, AP Exam scores, or transfer credit. (Students can revise their choice during the first two weeks of classes.) Table 1 shows data on the distributions of the first mathematics courses taken at MSU. The first four columns give the distributions for students in the Control group

[8]During the years of this study about 40% of incoming freshmen placed into College Algebra or the Tier 5 course.

who graduated in the years 1996-99; the next four columns give the corresponding distributions for Core-Plus students. Each column lists the percentages of students who took courses in Tiers 1–5 and the percent who took no mathematics course. The last four columns of Table 1 give the ratio CP/ Control of percents for each year and each tier. That ratio is a convenient way of comparing the two groups, and has the additional advantage that any year-to-year variations common to both groups cancel in the ratio. Because the Core-Plus high schools implemented the program in different years, what is relevent are *trends* in the course distribution over the years of the study.

Before looking at trends we note that in the initial year of the study the Core-Plus group was not statistically different from the Control group. Specifically, a χ^2 comparison of the first and fifth columns of Table 1 shows no statistically significant difference ($p = 0.57$ on 5 degrees of freedom); similarly there was no significant difference between the 1996 groups in ACT English or ACT Math scores ($p = 0.78$ and $p = 0.28$ respectively). Thus the 1996 Core-Plus group, while not obtained through a randomized study design, has a profile compatible with what one would expect from a random sample of the 1996 control population.

Table 1: **First courses at MSU— distribution by tiers**

	Control (Percent) (n= 2961)				CP (Percent) (n=353)				CP/Comp. Ratio			
Year	**96**	**97**	**98**	**99**	**96**	**97**	**98**	**99**	**96**	**97**	**98**	**99**
n	682	780	768	731	75	89	107	82				
Tier 1	18	20	24	21	15	13	9	6	0.80	0.67	0.39	0.29
Tier 2	13	12	16	18	12	10	5	9	0.94	0.77	0.29	0.48
Tier 3	18	15	14	12	20	13	13	10	1.08	0.91	0.92	0.80
Tier 4	29	27	26	27	36	27	39	33	1.26	1.00	1.50	1.20
Tier 5	19	22	15	19	17	33	28	38	0.89	1.45	1.81	1.95
None	2	4	4	2	0	4	6	5	0	1.13	1.35	2.10

Several trends are obvious in Table 1. In Tiers 1 and 2 there seems to be no significant year-to-year change in the Control group, but there is a clear downward trend in the Core-Plus group. The ratios show that in 1998 and 1999 students from Core-Plus high schools were taking Tiers 1 and 2 courses at only about 1/3 the rate of their Control group peers. In Tier 5 there is also little year-to-year change in the Control group, but an *upward* trend in the Core-Plus group. The ratios rise to the point where Core-Plus students are taking this remedial course at roughly twice the rate of the Control group students.

To determine whether these changes are statistically significant, the data in Table 1 were analyzed using a logistic regression model (for technical details about this model, see [1], Chapter 5 or [15], Section 8.4). The null hypothesis is that the data are drawn from a distribution that is constant in time. The model detects linear trends in the log-odds ratios of the corresponding entries of the column vectors of Table 1.

Here is a technical description of the model. Let X_{ij} be one of the 6×4 matrices that give the enrollments for the CP or the Control group (the actual enrollments, rather than the percentages as in Table 1). Let X_j be the j^{th} column and let n_j be the sum of the entries of X_j. We assume the columns are stochastically independent and that each column has a multinomial distribution,

$Y_j \sim \mathcal{M}_6\,(P_j = (p_{1j}, \ldots p_{6j}), n_j)$. Here, the p_{ij} are probabilities and $p_{1j} + \cdots + p_{6j} = 1$. The expected value of the j^{th} column is a vector $m_j = E(X_j) = n_j P_j$, and we let m be the 6×4 matrix $m = [m_1 \cdots m_4]$. Let μ be the 6×4 matrix obtained by taking the natural logarithm of m component-wise.

The model approximates μ as a linear combination

$$\mu = \log(m) \approx \beta_0\,\overline{I} + \sum_{i=1}^{5} \rho_i R_i + \sum_{j=1}^{3} \gamma_j C_j + \delta D$$

where $\overline{I}$ is the 6×4 matrix of all 1's, R_i is the 6×4 matrix whose i^{th} row is all 1's and all other entries are 0, C_j is the 6×4 matrix whose j^{th} column is all 1's and all other entries are 0, and D is the outer product $[0 \cdots 5]^T [1 \cdots 4]$ (which is a 6×4 matrix that represents the interaction between the rows and columns). The data determine estimates of the parameters $\beta_0, \rho_i, \gamma_j$ and δ. The number of real interest is the "log-odds-ratio" δ, which is a measure of the drift away from no change in the entries.

This model provides answers to three distinct questions: Does this model fit the data reasonably? Are the trends in the two sets of data significant? And how do the two sets of data compare? The results are as follows[9].

- For the Core-Plus data, the residual deviance (the χ^2 value on 14 degrees of freedom) is 12.99, indicating that the model fits these data very well. The estimate for δ is 0.134 with a z-value of 3.46. Thus the downward drift in the Core-Plus table is statistically significant ($p < .0005$).[10]
- For the Control group data, the residual deviance (χ^2 on 14 degrees of freedom) is 38.9, indicating that the model, while not fitting the data as well as in the previous case, is reasonable given that n, the number of students, is large. For these data the estimate for δ is $-.028$ with a z-value of -2.48. Thus there is a slight upward drift in the Control distribution that is statistically significant ($p = .0132$).
- Finally, the z-statistic for the comparison between the estimated drifts for the Core-Plus and the Control data is 3.97 ($p < .0001$)[11] .

Grades

We now turn from the courses students took to the grades they earned in those courses. As mentioned earlier, MSU mathematics department studies show that the Placement Exam is

[9] The calculations were done using the *glm* function in the statistical package S+.

[10] As mentioned in Section 2, we started with six Core-Plus schools and separated out two of them. At one of those schools, supplementation was done within Core-Plus courses and their non-Core-Plus courses were mainly for students not bound for college. If that school's data are included in Table 1, the residual deviance is 12.38, the estimate for δ is 0.097 with a z-value of 2.657, and the downward drift in the revised Core-Plus table is still statistically significant ($p < 0.008$).

[11] Let $\hat{\delta}_{CP}$ (resp. $\hat{\delta}_C$) denote the estimated drift for the CP (resp. Control) data, and let σ_{CP} (resp. σ_C) be the corresponding standard deviations. Then the z-statistic is

$$\frac{\hat{\delta}_{CP} - \hat{\delta}_C}{\sqrt{\sigma_{CP}^2 + \sigma_C^2}} \;\approx\; \frac{.1335 + .0276}{\sqrt{.039^2 + .0111^2}} \;\approx\; 3.97$$

corresponding to a p-value of .00007.

generally effective at placing students into courses appropriate for them. On the other hand, there have been claims that university placement exams underestimate the mathematical knowledge of students from reform curricula such as Core-Plus [12]. This latter contention suggests that the trends seen in Table 1 arise from shortcomings of the placement process rather than shortcomings in the preparation of Core-Plus students.

To determine the validity of the placement process, we examined the grades students attained in their first course at MSU. The first two columns of Table 3 list the average grades in the first MSU course taken by students in our sample who arrived at MSU in fall of 1998 and 1999 (we restrict attention to those years because some high schools did not adopt Core-Plus until fall, 1994). The last two columns give, for each tier, the z-statistic and p-value of the χ^2 test comparing the grade averages.

Table 2: **Average grades in the first MSU course taken by 1998-1999 students.**

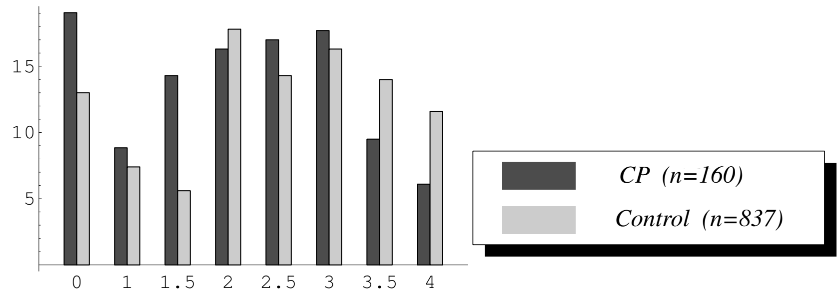
	Control	CP	z	p
Tier 1	2.79 (n=301)	3.42 (n=18)	-3.78	$< .0001$
Tier 2	2.86 (n=244)	2.38 (n=21)	1.62	0.11
Tier 3	2.36 (n=195)	2.15 (n=26)	0.99	0.32
Tier 4	2.29 (n=391)	1.94 (n=68)	2.25	.03
Tier 5	2.31 (n=251)	1.97 (n=66)	1.89	.06

For Tiers $2-5$, Table 2 shows that the Core-Plus students had consistently lower averages than their Control group peers. But except in Tier 4, those differences are not statistically significant at the .05 level. Thus the data on grades provides no evidence that the Core-Plus students were taking courses beneath their abilities. In fact, they suggest that the Core-Plus group may actually be doing somewhat worse in these courses.

The first row of Table 2 is quite different: the Core-Plus group definitely obtained better grades in Tier 1 courses. That is perhaps not surprising because the Tier 1 grades for the CP group in Table 2 are obtained from an increasingly selective group of students, changing from the top 15% in 1996 to the top 6% in 1999 (c.f. Table 1). A second possible factor is the effect of AP Calculus, which is often taken by top mathematics students; that point will be discussed in the next section. Nevertheless, Table 2 shows that there was an elite group of 18 students (an average of two per year from each Core-Plus school) who did extraordinarily well in some Tier 1 course at MSU. Interpreting that fact would require knowledge of which high school courses those students took.

It is illuminating to look at the grade distributions for the 1998-99 students who took Tier 3–5 courses (that includes roughly 90% of the Core-Plus students). For those students, the mean grade was 2.31 for the Control group and 1.99 for the Core-Plus group; that difference is statistically significant ($p = .0018$). The grade distributions, shown in Chart 3, make clear how that difference arises: the Core-Plus group has markedly fewer students who received grades of 3.5 and 4 and correspondingly more who received grades of 1.5 or less.

Chart 3: Grades received by 98-99 students in Tier 3–5 courses (percent receiving each grade).



Passing Calculus Eventually

As a third measure for judging the preparation of Core-Plus students for university-level mathematics, we looked at the percentages of students who eventually passed a Tier 1 course. At MSU, as at most colleges and universities, the first course in the Tier 1 technical calculus sequence is required for a wide range of engineering and physical science majors. It is a standard first-semester calculus course; essentially the same course is taught at a wide range of universities and in high school AP calculus classes. It requires a fluent working knowledge of algebra, geometry and trigonometry — the subjects of high school mathematics. The percent of students who pass Tier 1 calculus therefore provides a measure of the depth of high school mathematics preparation.

Our counts of those who "passed Tier 1 sometime" include students who received AP or transfer credit for calculus and those who passed a Tier 1 course as sophomores, juniors, or seniors at MSU, but does not include those who took the Tier 2 "business calculus" course. For comparison, during the years of this study about 25% of the freshmen who entered MSU eventually passed a Tier 1 calculus course.

Chart 4: Percent of students who passed a Tier 1 course sometime.

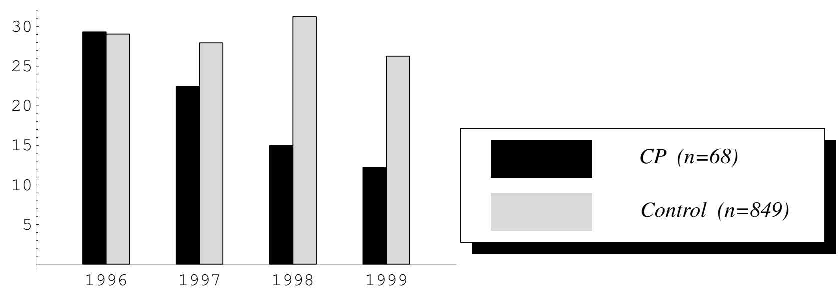


Chart 4 shows how the percentages of students who "passed Tier 1 sometime" changed over the four years of the study. The percentages for the Core-Plus group clearly declines. The best-fitting (maximum likelihood) line to the log-odds shows that the Core-Plus percentages fall at a rate of about 27% per year, and that decline is significant ($p < .005$).[12]

4 Other Aspects of the Data

This section presents data from several individual high schools. These were included in the composite data presented above, but looking at schools individually brings out several interesting phenomena not clearly visible in the previous analysis. In particular, we will uncover evidence

[12]The z-value is -2.84, corresponding to a p-value of .0046. The χ^2 statistic on five degrees of freedom is 7.07, which means that the model fits the data well.

that ACT scores and data from AP Calculus students need to be treated with care when used to evaluate high school mathematics programs.

Transition in One District

Two of the Core-Plus schools — the ones we denote CP4 and CP5 — are in the same district (in fact, they began as a single high school and moved to separate buildings during the years of our study). For these two schools we were able to obtain seven years of data, giving a large sample ($n = 254$) from a single district. These schools implemented Core-Plus Course 1 in the 1994-5 school year and subsequently added one course each year. Thus students who graduated in the years 1994-1997 had not used Core-Plus materials while some of the class of 1998, about half of the class of 1999, and most of the class of 2000 had taken Core-Plus courses. In the summer of 1999 this district's Board of Education reinstated a "traditional" track in mathematics as an option. As a result, the class of 2000 included about 30 students who had taken a special traditional precalculus class in their senior year.

Table 5: CP4-5: Average ACT and MSU Math Placement Exam Scores

HS class of	94–97	98–00	p value
ACT Engl	23.3	23.1	0.64
ACT Math	23.8	22.8	.06
Placement	12.6	11.5	0.10

Table 5 shows the average ACT and MSU Math Placement Exam scores of the 94-97 group and the 98-00 group; the last column shows the p-values of the χ^2 test comparing the groups. The very slight drop in ACT English scores is not significant, and the drops in ACT Mathematics and Placement Exam scores are only marginally significant. Thus the exam score data provide no firm statistical basis for predicting that the two groups would fare differently in their MSU mathematics courses.

Table 6: CP4-5: Distribution of first Math courses taken at MSU — Percent in each tier

HS class of	**94**	**95**	**96**	**97**	**98**	**99**	**00**
n	17	30	33	42	41	41	50
Tier 1	24	23	18	26	10	7	10
Tiers 2&3	41	40	42	36	27	24	32
Tiers 4&5	35	33	39	33	63	68	54
None	0	3	0	5	0	0	4
Passed Tier 1 sometime	35	43	36	45	12	12	20

The actual course data, given in Table 6, show a remarkable change in the distribution of the first mathematics courses taken at MSU by students from CP4 and CP5. There is an obvious drop-off in the percentage of students who make it into the Tier 1 courses and those who eventually passed a Tier 1 course and an obvious increase in the percentage placing into remedial and low-level algebra (Tiers 4&5). These differences are statistically significant[13].

Many studies of high school mathematics have used ACT or SAT scores as measures of students' preparation for college on the assumption that those scores are well-correlated to success in college mathematics courses. The data from CP4-5 suggest that study designs based on that assumption may fail to detect significant deficiencies in students' preparation for college-level mathematics — neither ACT scores nor Placement Exam scores predict the dramatic differences evident in the course data of Table 6.

A Much-Studied High School

One of the Core-Plus schools in our study, the one we denoted CP2, was the subject of two of the studies mentioned in our Literature Review. Those studies reached seemingly contradictory conclusions. Our data adds further information and allows us to formulate a hypothesis about why those studies reached different conclusions.

We begin by looking at the data from CP2. Table 7 shows the distribution of mathematics courses taken by CP2 students who came to MSU over a six-year period[14]. These distributions change abruptly between 1996 and 1997; there is again a sharp decrease in the percentage of students who took Tier 1–3 courses and those who passed Tier 1 eventually, and a corresponding increase in the percentages for Tiers 4 and 5.

Table 7: CP2: Distribution of first Math courses taken at MSU — Percent in each tier (n=185)

HS class of	**94**	**95**	**96**	**97**	**98**	**99**
n	25	39	26	34	40	21
Tier 1	16	10	12	3	3	5
Tiers 2 & 3	32	15	27	9	10	5
Tiers 4 & 5	52	67	62	82	78	76
None	0	8	0	6	10	14
Passed Tier 1 sometime	20	15	19	6	5	5

Tables 6 and 7 exhibit the same pattern: the level of courses taken declines abruptly between two consecutive years. The changes occur in different years, but in both cases it coincides with the first graduating class of Core-Plus students (the Core-Plus curriculum was begun with freshman at CP2 in 1993, a year earlier than at CP4-5). This is consistent with the hypothesis that the observed changes are attributable to the implementation of Core-Plus.

The patterns of ACT and MSU Math Placement scores, shown in Table 8, are also similar to those of CP4-5 in Table 5. However, Table 8 shows an increase in ACT English scores; that is keeping with a state and national trend, as pointed out by Milgram [8]. The ACT Math and MSU Placement Exam scores both decrease, but those declines again underestimate the downward trend seen in Table 7.

[13] The χ^2 statistic for a test of the null hypothesis of equal proportions among the five categories in Table 6 was 38.4 on four degrees of freedom, with corresponding p-value 9×10^{-8}. This large χ^2 value was caused primarily by the relative small frequencies in the Tier 1 and "Passed Tier 1 sometime" categories and the large frequencies in the Tier 4 & 5 category in the $98 - 00$ data.

[14] Although we obtained data for the class of 2000, we have not included it in this analysis because CP2 reinstated traditional mathematics courses as an option in fall of 1999.

Table 8: CP2: Average scores on ACT and MSU Math Placement Exam

HS class of	94	95	96	97	98	99
ACT English	21.5	23.1	22.8	23.6	23.7	25.3
ACT Math	23.6	22.5	21.7	20.7	20.5	22.4
Placement Exam	13.7	11.2	12.3	10.0	10.6	10.7

These conclusions about CP2 are consistent with the overall results described in Section 3 and with the conclusions of Bachelis and Milgram [8]. But they are *not* consistent with the Coxford data reported by Schoen and Hirsch in [11]. Why not?

Part of the explanation seems to lie in the distinct nature of the AP Calculus course and its students. AP Calculus is a highly structured curriculum designed to mimic a college calculus course, and is often taught by knowledgeable, experienced teachers to motivated, intellectually able students. Studies have shown that AP calculus students do well in subsequent mathematics courses [3], [4].

The Core-Plus students studied in Milgram [8] had not taken AP Calculus. In our study AP Calculus students constitute a small population which largely coincides with the Tier 1 group. In contrast, the Coxford study describes a group of CP2 graduates — those who placed into Precalculus, Calculus I, and three higher-level mathematics courses at the University of Michigan at Ann Arbor — many of whom had taken AP Calculus[15]. That is exactly the profile of the students who appear in our study in the anomalous Tier 1 row of Table 2. Thus our results *for those top students* are consistent with the Coxford study.

The apparent success of AP calculus students who go on to higher-level mathematics suggests that AP calculus might buffer the transition to college mathematics. Consequently, studies that use college mathematics data to evaluate high school mathematics curricula should treat AP calculus students as a distinct population.

Schools Supplementing Core-Plus

The four schools in our Core-Plus group chose to adopt Core-Plus as their sole mathematics curriculum and to eliminate their previous curriculum, and two of them had completed this process by 1998-99. That purity seems to be rare; at least one study of the Core-Plus program has noted that "Most Core-Plus high schools maintain two different mathematics programs, Core-Plus and a more traditional one" [7]. Amongst such schools there is no consistency in the options available: some have an alternative track, some offer various algebra and precalculus courses specifically aimed at preparing students for college mathematics, and some incorporate additional algebra and precalculus material directly into their Core-Plus courses.

As mentioned earlier, two high schools in our sample fit this "Supplemented Core-Plus" profile. The information on their students was therefore not included in the data examined in Section 3 (except for footnote 10). Both schools are relatively small and the data on the MSU courses taken by their students is remarkably inconsistent — see Table 9. It is perhaps notable

[15] Of the five University of Michigan courses in the Coxford study, all but the lowest correspond to Tier 1 courses at MSU, and freshmen in the three higher courses necessarily had AP Calculus credit. The Coxford study looked only at those students who took mathematics in their freshman year at college and included grades from both first and second semester courses.

that the clear declines seen in the data from the four CP schools are not evident here. However, this data set is simply insufficient to draw any conclusions.

Table 9: Distribution (by percent) of first Math courses for the "Supplemented Core-Plus" schools (n=103).

HS class of	96	97	98	99
Tier 1	19	38	43	15
Tiers 2 & 3	31	10	25	22
Tiers 4 & 5	50	52	32	63
Passed Tier 1 sometime	27	43	54	19

5 Implications for Studies Evaluating New Curricula

The methodology and results of this study raise a number of issues relevant for future studies of curricula. We describe those issues in this brief section. The observations made here have implications both for study design and for the interpretation of studies of high school programs.

Our first point concerns the manner in which new curricula are evaluated. It is too easy for partisans of a new curriculum to find (perhaps even correctly) that the new curriculum succeeds at the goals set for it, and miss consequences that show up at the next level. Thus:

- *Evaluations of new curricula should contain follow-up studies at the next level of curriculum.* Ideally, those follow-up studies should be done by independent researchers.

We also have four observations which may help in designing reliable studies of how well high school programs prepare students for college courses.

- *ACT exam scores should not be relied upon by themselves to draw conclusions about the effectiveness of high school curricula.* Most students take ACT exams during their junior year of high school, and the exams themselves involve material below junior level. Our results indicate that, at least in some circumstances, ACT scores fail to detect (or to fully reflect) great changes in student mathematics learning.
- *Evaluations of high school curricula should be careful to treat AP Calculus students as a distinct group.* As noted in [3] and [4], AP Calculus prepares students well for college mathematics by providing a year of highly structured, demanding mathematics which is largely independent of the mathematics program their high school is using.
- *Our study was feasible because we focused on schools that were implementing a single curriculum.* The schools in our study began with an entirely non-Core-Plus curriculum and moved, over the years studied, steadily toward one that offered Core-Plus as students' only mathematics option (except for AP Calculus). We then applied a statistical analysis that looked for trends over time in those students' college mathematics achievement. This analysis did not require knowledge of each student's high school courses, and did not hinge

on complete implementations of the new curriculum. In contrast, studies involving mixed-curricula schools would need specific high school course data on every student and would have to address many confounding issues such as how students selected courses, what the precise content of those courses was, what kind of 'tracking' occurred, etc.

- *Studies which compare school populations before and after whole-school implementation of a new program avoid many of the issues associated with extraneous variables.* Our analysis compares students who had no exposure to the Core-Plus program with Core-Plus students who attended *the same school* a few years later. These two groups have the same demographics, lived in the same community, attended the same high school, and often were taught by the very same teachers.

From a statistical viewpoint, one would ideally want randomized experiments to disentangle the effects of the curriculum from other potential explanations. The last point above suggests that the method of analysis used in this study goes a long way toward eliminating all variables except those directly associated with the implementation of the Core-Plus program, such as teacher training immediately before or during implementation, and associated changes in students' elementary and middle school mathematics preparation. To assess the effects of those implementation-specific variables, one would like to see corroborating studies involving a large number of high schools. Our work suggests that such large-scale studies assessing outcomes from high school programs can be done using university data.

6 Conclusion

The effectiveness of Core-Plus and the other new NSF-funded curricula high school programs is a significant issue for college mathematics faculty. High schools across the U.S. are adopting these programs in the hope of improving their students' understanding of mathematics. Increasing numbers of students are arriving on campus having learned mathematics using these programs.

There is considerable evidence for the shortcomings of the entire mix of mathematics curricula currently used in American high schools. Our own studies of Michigan high schools revealed an eclectic assortment of high school courses, and international studies show that the US curriculum as a whole lacks the depth and coherence seen in the curricula of countries with top-performing mathematics and science students. One would hope that a new high school mathematics program would produce students who are at least as mathematically able as those coming out of the previous program. We undertook the present study to determine whether Core-Plus does that, and to determine if large scale studies are warranted.

The context of our study is limited. Our data include only students who enrolled at MSU. We did not examine effectiveness of Core-Plus for students who did not go to college, nor did we reach any conclusion about high school curricula which mix Core-Plus courses with more standard courses. But our data come from a broad group of college students, and the statistical conclusions are robust and consistent.

The data show a clear decline in the level of Michigan State University mathematics courses taken by Core-Plus graduates. The existence of that decline is statistically significant at any reasonable level. The decline in course level is accompanied by a decline in average grades for all but the very top students, and a decline in the percentages of those who eventually passed a technical calculus course. These trends occur in data that include students from a variety of communities. The data from individual high schools show that the timing of these declines corresponds precisely with the implementation of the Core-Plus program.

While the attribution of causality is impossible in this study, the results are compelling. Except for some top students, graduates of Core-Plus mathematics are struggling in college mathematics at Michigan State University. The evidence shows that they were less well prepared than both graduates in the Control group and graduates of their own high school before the implementation of Core-Plus mathematics. At the very least, these results point the need for larger and broader studies of how Core-Plus students fare in college mathematics.

From a broader perspective, there is a need for high-quality statistical studies of how well high school mathematics programs *of all kinds* are preparing students for college mathematics. Such studies would provide vital information for high school administrators and teachers when they choose new mathematics programs and textbook series. The fact that such studies are not routinely done points to disconnects and lapses on several fronts: insufficient communication between high school and university mathematics departments, a failure of mathematics education professionals to gather hard data on the success of current high school mathematics programs, and a failure of funding agencies to encourage such activities. We hope that the NSF would fund such studies (and require them for NSF-funded curricula) and that math faculty — who see the effects of high schools curricula in their daily teaching — will recognize the importance of participating in them.

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