### Definitions

- A sequence \( \{a_n\}_{n=1}^\infty \) is an ordered list:
  \[ a_1, a_2, a_3, a_4, \ldots \]

- A series \( \sum_{n=1}^\infty a_n \) is the sum of a sequence:
  \[ a_1 + a_2 + a_3 + a_4 + \cdots \]

- A “p-series” is one of the form \( \sum_{n=1}^\infty \frac{1}{n^p} \)

- A “geometric series” is one of the form \( \sum_{n=1}^\infty r^n \)

- An “alternating series” is one of the form \( \sum (-1)^n a_n \) or \( \sum (-1)^{n-1} a_n \)

### Important Formulas

- If \( |r| < 1 \), then the geometric series \( \sum_{n=0}^\infty r^n \) converges to \( \frac{1}{1-r} \). If \( |r| \geq 1 \), then the series diverges.

### Convergence Tests

- **n\(^{th}\) Term Test:** If \( \lim_{n \to \infty} a_n \neq 0 \), then \( \sum a_n \) diverges.

- **Integral Test:** If \( a_n = f(n) \), where \( f(x) \) is continuous, positive, and decreasing on \([1, \infty)\), then \( \sum a_n \) converges if and only if \( \int_1^\infty f(x)dx \) converges.

- **p-Series Test:** If \( \sum a_n \) is a p-series with \( p > 1 \), then it converges. If \( p \leq 1 \), then it diverges.

- **Direct Comparison Test:** If \( a_n \) and \( b_n \) are non-negative, with \( a_n \leq b_n \) for all \( n \), then \( \sum b_n \) diverges if \( \sum a_n \) diverges, and \( \sum a_n \) converges if \( \sum b_n \) converges.

- **Limit Comparison Test:** If \( a_n \) and \( b_n \) are non-negative, and \( \lim_{n \to \infty} \frac{a_n}{b_n} = c \) for some constant \( c \) in \((0, \infty)\), then \( \sum a_n \) converges if and only if \( \sum b_n \) converges.

- **Alternating Series Test:** An alternating series converges if the sequence of terms is decreasing and converges to zero.

- **Absolute Convergence Test:** If \( \sum |a_n| \) converges, then \( \sum a_n \) converges.

- **Ratio Test:** If \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1 \), then \( \sum a_n \) converges (absolutely), and if \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| > 1 \), then \( \sum a_n \) diverges.