Show your work (not just final answers).

1. Consider the region in the plane bounded by the curves \( y = e^x \), \( x = 0 \), and \( x = 1 \).

   (a) (3 points) Find the volume of the solid generated by rotating this region around the \( x \)-axis.

   **Solution:** Each cross section is a circle with radius \( e^x \), and so area \( A(x) = \pi e^{2x} \).
   The volume is then
   \[
   V = \pi \int_0^1 e^{2x} \, dx = \frac{\pi}{2} \left[ e^{2x} \right]_0^1 = \frac{\pi}{2} (e^2 - 1)
   \]

   (b) (3 points) Find the volume of the solid generated by rotating this region around the line \( y = -1 \).

   **Solution:** Each cross section is a washer with inner radius 1 and outer radius \( e^x + 1 \), and so the area is
   \[
   A(x) = \pi \left( (e^x + 1)^2 - 1 \right) = \pi \left( e^{2x} + 2e^x \right)
   \]
   The volume is then
   \[
   V = \pi \int_0^1 (e^{2x} + 2e^x) \, dx
   = \pi \left( \frac{1}{2} \left[ e^{2x} \right]_0^1 + 2 \left[ e^x \right]_0^1 \right)
   = \pi \left( \frac{1}{2} (e^2 - 1) + 2(e - 1) \right)
   = \frac{\pi}{2} (e^2 + 4e - 5)
   \]

Continue on to back side
2. (4 points) A conical tank has base radius 1 meter and height 3 meters. The tank is full of liquid that weighs 5 Newtons per cubic meter. How much work is required to pump all of the liquid to the top of the tank?

**Solution:** The work is calculated with the integral:

\[ W = 5 \int_0^3 (3 - y)A(y)dy, \]

where \( A(y) \) is the cross-sectional area of the tank. Since it is a cone, \( A(y) = \pi \left( \frac{y}{3} \right)^2 = \frac{\pi}{9} y^2 \). So we get

\[
\begin{align*}
W &= \frac{5\pi}{9} \int_0^3 (3y^2 - y^3)dy \\
&= \frac{5\pi}{9} \left[ y^3 - \frac{1}{4} y^4 \right]_0^3 \\
&= \frac{5\pi}{9} \left( 27 - \frac{81}{4} \right) \\
&= \frac{15\pi}{4}
\end{align*}
\]