There are 6 problems on 5 pages.

1. Clear everything from your desk except writing utensils, pencil sharpeners, and erasers. You are not allowed to have calculators, cell phones, or any other electronic devices out.

2. Please turn your cell phone off or to vibrate.

3. Unless explicitly stated in the problem (or if the problem is multiple choice, true/false, or fill-in-the-blank), you will be graded on the work you show as well as your final answer.
1. (24 points) Differentiate the following functions:

(a) \( f(x) = \frac{\sin(x)}{x^2 + 1} \)

(b) \( f(x) = \left(x^2 - \frac{1}{x}\right)^2 \)

(c) \( f(x) = 2 \cdot \sqrt{1 - \cos(x)} \)

(d) \( f(x) = (x - 3)^2(2x - 5)^3 \)
2. (16 points) Below is a curve called the “Folium of Descartes”, defined by the implicit equation:

\[ x^3 + y^3 = 3xy \]

Find equations for both the **tangent line** and the **normal line** to the curve at the point \((3/2, 3/2)\), which has been labelled in the picture. (Show work to support your answers)
3. (16 points) Let \( p(x) \) be the quadratic polynomial \( p(x) = x^2 + ax + b \), and define the piecewise function on the domain \((0, \infty)\):

\[
f(x) = \begin{cases} 
\frac{1}{x} & \text{if } 0 < x \leq \frac{1}{2} \\
p(x) & \text{if } x > \frac{1}{2}
\end{cases}
\]

Find values for \( a \) and \( b \) that make \( f(x) \) continuous and differentiable for all \( x > 0 \).

4. (12 points) Recall (from algebra/precalculus) that an **even function** is a function where \( f(-x) = f(x) \) for all \( x \), and an **odd function** is one where \( f(-x) = -f(x) \) for all \( x \). Prove that the derivative of any odd function is an even function.
5. (16 points) Take two line segments of fixed lengths $a = \sqrt{3}$ and $b = 2$, which meet at a point. Consider the triangle formed by joining the other two endpoints, and let $c$ denote the length of the third side. As the angle $\theta$ between the segments changes, the length, $c$, changes with it. If $\theta$ is increasing (meaning side $b$ is rotating counter-clockwise) at a constant rate of 5 radians per minute, then what is the rate at which the length $c$ is changing when $\theta = \frac{\pi}{6}$?

(Hint — Recall the law of cosines: $c^2 = a^2 + b^2 - 2ab \cos(\theta)$)
6. (16 points) You are standing on the edge of the roof of a 100-ft tall building. You throw a tennis ball upward and off the edge, with an initial upward velocity of 6 feet per second. The height of the ball above the ground (in feet) after \( t \) seconds is given by

\[
h(t) = 100 + 6t - 16t^2
\]

(a) At what time is the velocity of the ball equal to zero?

(b) At what time is the ball falling at a rate of 6 ft/s?

(c) What is the acceleration of the ball at \( t = 1 \) sec?

(d) Given that the ball hits the ground after about 3 seconds, estimate how fast the ball is moving right before it hits the ground.

<table>
<thead>
<tr>
<th>Problem</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points Earned</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Possible 24</td>
<td>16</td>
<td>16</td>
<td>12</td>
<td>16</td>
<td>16</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>