Clear your desk of everything except pens, pencils and erasers. **Show all your work.**

If you have a question raise your hand and I will come to you.

1. (1 point) **Multiple Choice. Circle the best answer. No partial credit available**

   Find the derivative of \( f(t) = \frac{\tan t - 1}{\sec t} \) at \( t = \pi \).

   A. \( f'(\pi) = -1 \)
   
   B. \( f'(\pi) = 0 \)
   
   C. \( f'(\pi) = 1 \)
   
   D. \( f'(\pi) = 2 \)
   
   E. None of the above.

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**Solution:** You can use the quotient rule, using the derivatives of \( \tan(t) \) and \( \sec(t) \):

\[
f'(t) = \frac{\sec^2(t) \cdot \sec(t) - (\tan(t) - 1) \sec(t) \tan(t)}{\sec^2(t)} = \frac{\sec^3(t) - (\tan(t) - 1) \sec(t) \tan(t)}{\sec^2(t)}
\]

Then just plug in \( t = \pi \) to get

\[
f'(\pi) = \frac{(-1)^3 - (0 - 1)(-1)(0)}{(-1)^2} = -1
\]

An easier way however, is to first simplify the expression by writing \( \tan(t) \) as \( \frac{\sin(t)}{\cos(t)} \) and \( \sec(t) \) as \( \frac{1}{\cos(t)} \), so we can also write \( f(t) \) as

\[
f(t) = \sin(t) - \cos(t)
\]

Then differentiating, we get

\[
f'(t) = \cos(t) + \sin(t)
\]

Plugging in \( t = \pi \), we get

\[
f'(\pi) = -1 + 0 = -1
\]
2. (1 point) **Fill-in-the-Blank. No partial credit available**

Suppose \( f \) and \( g \) are functions of \( x \) that are differentiable at \( x = 1 \) and that

\[
\begin{align*}
  f(1) &= 7 & f'(1) &= -5 & g(1) &= -4 & g'(1) &= 2 \\
\end{align*}
\]

(a) \[
\frac{d}{dx} \left( fg \right) \bigg|_{x=1} = f'(1) \cdot g(1) + f(1) \cdot g'(1) = 34 \\
\]

(b) \[
\frac{d}{dx} \left( \frac{f}{g} \right) \bigg|_{x=1} = \frac{f'(1) \cdot g(1) - f(1) \cdot g'(1)}{g(1)^2} = \frac{3}{8} \\
\]

(c) \[
\frac{d}{dx} \left( 2g - 3f \right) \bigg|_{x=1} = 2 \cdot g'(1) - 3 \cdot f'(1) = 19 \\
\]
3. (1 point) Find the derivative of \( f(x) = \sin \left( \sqrt{\frac{1}{x+2}} \right) \)

**Solution:** If we denote \( g(x) = \sin(x), h(x) = \sqrt{x}, \) and \( k(x) = \frac{1}{x+2}, \) then we can write \( f \) as the composition \( f = g \circ h \circ k. \) Then the chain rule says:

\[
 f' = g'(h(k)) \cdot h'(k) \cdot k'
\]

We know the derivatives of \( g, h, \) and \( k: \)

\[
 g' = \cos(x) \\
 h' = \frac{1}{2\sqrt{x}} \\
 k' = \frac{-1}{(x+2)^2}
\]

So we have that

\[
 f' = \cos \left( \sqrt{\frac{1}{x+2}} \right) \cdot \frac{1}{2\sqrt{\frac{1}{x+2}}} \cdot \frac{-1}{(x+2)^2}
\]

4. (1 point) Use the quotient rule (and the fact that \( \cot(x) = \frac{\cos(x)}{\sin(x)} \)) to show that

\[
 \frac{d}{dx}(\cot(x)) = -\csc^2(x)
\]

**Solution:**

\[
 \frac{d}{dx}(\cot(x)) = \frac{d}{dx} \left( \frac{\cos(x)}{\sin(x)} \right) \\
 = \frac{d}{dx}(\cos(x)) \cdot \sin(x) - \cos(x) \cdot d\frac{\sin(x)}{dx} \\
 = \frac{-\sin(x)^2 - \cos(x)^2}{\sin(x)^2} \\
 = \frac{-(\sin(x)^2 + \cos(x)^2)}{\sin(x)^2} \\
 = \frac{-1}{\sin(x)^2} \\
 = -\csc^2(x)
\]