5 Curve Sketching

Remark 5.1 (General Guidelines for Curve Sketching).
When trying to sketch the graph of a given function $f(x)$, ask yourself the following:

1. What is the domain of $f$?
2. What are the $x$ and $y$ intercepts of the graph?
   (a) Evaluate $f(0)$ for $y$-intercept
   (b) Solve $f(x) = 0$ for $x$-intercept
3. Does the graph have any symmetry (is $f$ an odd function? an even function? neither?)
   (a) Even if $f(-x) = f(x)$ for all $x$
   (b) Odd if $f(-x) = -f(x)$ for all $x$
4. Does the graph have any asymptotes (horizontal, vertical, slant)?
   (a) Has horizontal asymptote $y = c$ if $\lim_{x \to \infty} f(x) = c$ or if $\lim_{x \to -\infty} f(x) = c$.
   (b) Has vertical asymptote $x = c$ if either of $\lim_{x \to c^+} f(x)$ or $\lim_{x \to c^-} f(x)$ are equal to either $\infty$ or $-\infty$.
   (c) Has slant asymptote $y = mx + b$ if $\lim_{x \to \infty} (f(x) - (mx + b)) = 0$ or if $\lim_{x \to -\infty} (f(x) - (mx + b)) = 0$.
5. Where is $f(x)$ increasing and decreasing?
   (a) See Theorem 3.1
6. Where are the local maximum and minimum points?
   (a) Use First Derivative Test or Second Derivative Test
7. Where is the graph concave up and where is it concave down?
   (a) See Theorem 3.12
8. Where are the inflection points?
   (a) Find where $f''(x)$ changes sign

Example 5.2 (Instructor).
Sketch graphs of the following:
(a) $y = x(x - 4)^3$
(b) $y = \frac{x^3}{x^2 + 1}$
(c) $y = \frac{x}{\sqrt{x^2 + 1}}$
(d) $y = \sin^3(x) = (\sin(x))^3$

Example 5.3 (Student).
Sketch graphs of the following:
(a) $y = x^5 - 5x$
(b) $y = \frac{x}{x - 1}$
(c) $y = 1 + \frac{1}{x} + \frac{1}{x^2}$
(d) $y = x + \cos(x)$