3 Differentiation Formulas

Theorem 3.1 (Constant, Linear, Power).
Take \( c \) and \( n \) to be any constants
\[
\frac{d}{dx} (c) = 0 \quad \frac{d}{dx} (x) = 1 \quad \frac{d}{dx} (x^n) = nx^{n-1}
\]

Theorem 3.2 (Constant Multiple, Sum, Difference).
Take \( c \) to be a constant and \( f, g \) both differentiable functions, then:
\[
\frac{d}{dx} (cf(x)) = cf’(x) \quad \frac{d}{dx} (f(x) + g(x)) = f’(x) + g’(x) \quad \frac{d}{dx} (f(x) - g(x)) = f’(x) - g’(x)
\]

Theorem 3.3 (Product, Quotient).
Take \( f, g \) both differentiable functions, then:
\[
\frac{d}{dx} (f(x)g(x)) = f’(x)g(x) + f(x)g’(x) \quad \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f’(x)g(x) - f(x)g’(x)}{|g(x)|^2}
\]

Remark 3.4.
The most powerful theorem we currently have is \( \frac{d}{dx} (x^n) = nx^{n-1} \) (the power rule... pun intended). You will see that it gets used in nearly every problem. The proof of it is very complicated and is typically broken down into steps
1. \( n \) is a positive integer (proof is in this section, need binomial theorem)
2. \( n \) is a negative integer (proof in this section, need quotient rule)
3. \( n \) is a rational number (need implicit differentiation, 2.6)
4. \( n \) is any real number (need knowledge of logarithmic differentiation, MTH133)

4 Derivatives of Trigonometric Functions

Theorem 4.1.
\[
\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \lim_{x \to 0} \frac{\cos x - 1}{x} = 0
\]

Theorem 4.2.
If \( \lim_{x \to a} f(x) = 0 \) then
\[
\lim_{x \to a} \frac{\sin(f(x))}{f(x)} = 1
\]

Proof \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \)

http://youtu.be/Ve99biD1KtA
**Example 4.3** (Instructor).

Differentiate \( \frac{x^2 + 2x - 3}{\sqrt{x}} \)

**Example 4.4** (Instructor).

Find the equation of the tangent line to the curve \( y = \frac{2x}{x+1} \) at the point (1, 1).

**Example 4.5** (Instructor).

For what values of \( x \) does \( f(x) = x^3 + 3x^2 + x + 3 \) have a horizontal tangent line?

**Example 4.6** (Instructor).

Find the limit \( \lim_{x \to 0} \frac{\sin 5x}{x} \)

**Example 4.7** (Instructor).

Find the limit \( \lim_{x \to 0} \frac{\sin 2x}{x^2 + x} \)

**Example 4.8** (Instructor).

Find the limit \( \lim_{x \to 0} \frac{\tan 3t}{\sin t} \)

**Example 4.9** (Students).

Find the velocity and acceleration of the position function \( s(t) = \frac{3}{2 - x} \)

**Example 4.10** (Students).

Find the equations of the tangent lines to the curve \( y = \frac{x - 1}{x + 1} \) that are parallel to the line \( x - 2y = 2 \)

**Example 4.11** (Students).

Let \( f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ mx + b & \text{if } x > 2 \end{cases} \) Find the values of \( m \) and \( b \) that make \( f \) differentiable everywhere.

**Example 4.12** (Students).

Find the limit \( \lim_{x \to 0} \frac{\cos x - 1}{\sin x} \)

**Example 4.13** (Students).

Find the limit \( \lim_{x \to 0} \frac{\sin(x^2)}{x} \)

**Example 4.14** (Students).

Find the limit \( \lim_{x \to 1} \frac{\sin(x - 1)}{x^2 + x - 2} \)