1. (5 points) Sketch a graph of the function \( f(x) = \frac{6x-1}{2x+10} \). You may assume that the derivatives are:

\[
\begin{align*}
  f'(x) &= \frac{31}{2(x+5)^2} \\
  f''(x) &= \frac{-31}{(x+5)^3}
\end{align*}
\]

(a) What are the horizontal and vertical asymptotes?

The degree of the numerator and denominator are the same. So the horizontal asymptote is obtained by dividing the leading coefficients. So it is \( y = \frac{6}{2} = 3 \). The vertical asymptotes occur where the denominator is zero, so at \( x = -5 \).

(b) Where is \( f(x) \) increasing? Where is \( f(x) \) decreasing?

\( f'(x) \) is always positive, so \( f(x) \) is increasing on its whole domain, which is \(( -\infty, -5) \cup (-5, \infty) \). It is not decreasing anywhere, since \( f'(x) \) is never negative.

(c) Where is \( f(x) \) concave up? Where is \( f(x) \) concave down?

\( f''(x) \) is positive when \( x < -5 \) and is negative when \( x > -5 \). So the graph of \( f(x) \) is concave up on \(( -\infty, -5) \) and concave down on \(( -5, \infty) \).

(d) Finally, sketch the graph of \( f(x) \).
2. (5 points) For any $x$ in $[0,1]$, draw a right triangle under the graph of $y = x^3 - x$ with hypotenuse connecting the origin to the point $(x, x^3 - x)$ on the graph, as shown in the picture. Which value of $x$ will give the triangle with the largest area?

Since the triangle goes below the $x$-axis in the interval $[0,1]$, the height of the triangle is actually $-(x^3 - x) = x - x^3$, so the area is given by $A = \frac{1}{2} x \cdot (x - x^3) = \frac{1}{2} (x^2 - x^4)$. Differentiating, we get $A' = \frac{1}{2} (2x - 4x^3) = x(1 - 2x^2)$. From this, we see that $A' = 0$ at $x = 0$ and $x = \frac{1}{\sqrt{2}}$. But $A(0) = 0$, so that will not be the maximum, so it seems like it will be at $x = \frac{1}{\sqrt{2}}$. Technically, we should check that this is, in fact, a local maximum. The second derivative is $A'' = 1 - 6x^2$, which is negative at $x = \frac{1}{\sqrt{2}}$, so yes, this is a local maximum, and it is the global maximum on $[0,1]$. 