1. (4 points) Fill in the blanks:

Consider the function \( f(x) = \frac{1}{x} \) on the domain \([\frac{1}{2}, 1]\). Since \( f(x) \) is “\textbf{continuous}” on \([\frac{1}{2}, 1]\) and “\textbf{differentiable}” on \((\frac{1}{2}, 1)\), the \textbf{Mean Value Theorem} says that there is some number \( c \) in \((\frac{1}{2}, 1)\) with \( f'(c) = -2 \).

2. (2 points) Consider the rational function

\[
f(x) = \frac{4x^3 - 8x^2 + 5x - 7}{3(x - 1)^3}
\]

(a) Identify all vertical asymptototes of \( f(x) \).

\textbf{Solution.} The vertical asymptotes occur where the denominator is zero. So \( x = 1 \) is the only vertical asymptote.

(b) Identify all horizontal asymptotes of \( f(x) \).

\textbf{Solution.} The numerator and denominator have the same degree, so the horizontal asymptote is the quotient of the leading coefficients (4 on top, 3 on bottom). The only asymptote is \( y = \frac{4}{3} \).
3. (4 points) Below is the graph of the derivative of a function $f(x)$.

(a) What are the critical points of $f(x)$?

**Solution.** The critical points of $f(x)$ are where $f(x)'$ is zero. So they are at the $x$-intercepts of the graph of $f'(x)$, which we can see are $x = -1$ and $x = 1$.

(b) Tell whether each critical point from part (a) is a local minimum, local maximum, or neither.

**Solution.** Use the First Derivative Test. At $x = -1$, $f'(x)$ goes from negative to positive, so it is a local minimum. At $x = 1$, $f'(x)$ does not change sign, so it is neither a max or a min.