1. Calculate the derivatives of the following functions:

(a) \( f(x) = \sin^2(3x) \)

**Solution.** Use the chain rule. If \( F(x) = x^2, G(x) = \sin(x), \) and \( H(x) = 3x, \) then our function can be written as \( f = F \circ G \circ H. \) So the chain rule says

\[
\frac{df}{dx} = F'(G(H(x))) \cdot G'(H(x)) \cdot H'(x)
\]

The derivatives are \( F'(x) = 2x, G'(x) = \cos(x), \) and \( H'(x) = 3. \) So we get

\[
\frac{df}{dx} = 2 \sin(3x) \cos(3x) \cdot 3 = 6 \sin(3x) \cos(3x)
\]

(b) \( f(x) = \frac{\sqrt{x} \tan(x)}{x^4 + x^2 + 1} \)

**Solution.** Use the quotient rule. If \( F(x) = \sqrt{x} \cdot \tan(x) \) and \( G(x) = x^4 + x^2 + 1, \) then \( f = \frac{F}{G}, \) and so

\[
\frac{df}{dx} = \frac{F'(x) \cdot G(x) - F(x) \cdot G'(x)}{G(x)^2}
\]

The derivative of \( G \) is easy: just use the power rule and get \( G'(x) = 4x^3 + 2x. \) For the derivative of \( F, \) use the product rule:

\[
F'(x) = \frac{d}{dx}(\sqrt{x}) \cdot \tan(x) + \sqrt{x} \cdot \frac{d}{dx}(\tan(x)) = \frac{1}{2\sqrt{x}} \tan(x) + \sqrt{x} \sec^2(x)
\]

So all together, we get

\[
\frac{df}{dx} = \left( \frac{1}{2\sqrt{x}} \tan(x) + \sqrt{x} \sec^2(x) \right) (x^4 + x^2 + 1) - \sqrt{x} \tan(x)(4x^3 + 2x)
\]

\[
= \frac{(x^4 + x^2 + 1)^2}{x^4 + x^2 + 1}
\]

2. Evaluate the limit

\[
\lim_{x \to 0} \frac{\sin(x)}{\sqrt{x}}
\]

**Solution.**

\[
\lim_{x \to 0} \frac{\sin(x)}{\sqrt{x}} = \lim_{x \to 0} \left( \frac{\sin(x)}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} \right)
\]

\[
= \lim_{x \to 0} \left( \frac{\sin(x)}{x} \cdot \sqrt{x} \right)
\]

\[
= \left( \lim_{x \to 0} \frac{\sin(x)}{x} \right) \cdot \left( \lim_{x \to 0} \sqrt{x} \right)
\]

\[
= 1 \cdot 0
\]

\[
= 0
\]
3. A car is driving down a straight road, and its distance (in miles) from the starting point at time \( t \) (in minutes), for \( 0 \leq t \leq 4 \), is given by the function

\[ s(t) = \sqrt{t} - \frac{t^2}{8} \]

(a) What is the velocity (in miles per hour) at time \( t = 1 \) minute?

**Solution.** The velocity at time \( t \) is given by \( s'(t) = \frac{1}{2\sqrt{t}} - \frac{t}{4} \). So at \( t = 1 \), we have \( s'(1) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \) miles per minute.

(b) When is the car’s velocity zero?

**Solution.** Take the velocity function we got earlier, set it equal to zero, and solve for \( t \) (this is actually just algebra; no calculus involved).

\[
\begin{align*}
\frac{1}{2\sqrt{t}} - \frac{t}{4} &= 0 \\
\frac{1}{2\sqrt{t}} &= \frac{t}{4} \\
\frac{2}{\sqrt{t}} &= t \\
\frac{4}{t} &= t^2 \\
t^3 &= 4 \\
t &= \sqrt[3]{4}
\end{align*}
\]