1. Calculate the derivatives of the following functions:

(a) \( f(x) = (2x + 3)(x^3 + 3x^2 + 4x + 5) \)

**Solution.** The product rule says that \( \frac{d}{dx}(gh) = g' \cdot h + g \cdot h' \). In this particular case, use \( g = 2x + 3 \) and \( h = x^3 + 3x^2 + 4x + 5 \). Then \( g' = 2 \) and \( h' = 3x^2 + 6x + 4 \). So we get

\[
f'(x) = 2(x^3 + 3x^2 + 4x + 5) + (2x + 3)(3x^2 + 6x + 4)
\]

(b) \( f(x) = \frac{2x^3 + 3x^2 + 4x + 5}{x^3 + 3x^2 + 4x + 5} \)

**Solution.** The quotient rule says that \( \frac{d}{dx} \left( \frac{g}{h} \right) = \frac{g'h - gh'}{h^2} \). In this particular case, use \( g = 2x + 3 \) and \( h = x^3 + 3x^2 + 4x + 5 \), just as in part (a). So we get

\[
f'(x) = \frac{2(x^3 + 3x^2 + 4x + 5) - (2x + 3)(3x^2 + 6x + 4)}{(x^3 + 3x^2 + 4x + 5)^2}
\]

2. What is the equation of the tangent line to the graph of \( f(x) = 3x^2 + 7x - 2 \) at the point \( x = 1 \)?

**Solution.** The slope of the tangent line at \( x = 1 \) is given by \( f'(1) \). So we must first compute the derivative:

\[
f'(x) = 6x + 7
\]

So the slope of the tangent line is given by \( f'(1) = 6(1) + 7 = 13 \). Since the tangent line must go through the point \( (1, f(1)) = (1, 8) \), we can just use the point-slope formula for a line to get

\[
y - 8 = 13(x - 1)
\]
3. Calculate $f'(x)$ if $f(x) = \frac{1}{x^2}$ using the definition of the derivative. 
(You must do the limit computation to receive credit)

**Solution.** Use the limit definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+2}{(x+2)(x+h+2)} - \frac{x+h+2}{(x+2)(x+h+2)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h(x+2)(x+h+2)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+2)(x+h+2)}$$

$$= -\frac{1}{(x+2)^2}$$