1. (2 points)

(a) Evaluate the sum $\sum_{k=1}^{3} (k + 1)$.

$$\sum_{k=1}^{3} (k + 1) = (1 + 1) + (2 + 1) + (3 + 1) = 2 + 3 + 4 = 9.$$ 

(b) Write the following sum in sigma notation:

$$\frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{8} \cdot 4 + \frac{1}{10} \cdot 5 = \sum_{k=1}^{5} \frac{1}{2k}$$

2. (4 points) Estimate the area under the graph of $f(x) = x^2 + 1$ from $x = 0$ to $x = 3$ with a left-hand sum using 3 sub-intervals of equal width.

The image below shows the rectangles used in the left-hand sum.

The areas of these rectangles are 1, 2, and 5, so the total estimated area is $1 + 2 + 5 = 8.$
3. (4 points) Suppose that \( f(x) \) and \( g(x) \) are functions with the following definite integrals:

\[
\begin{align*}
\int_0^7 f(x) \, dx &= 9 & \int_7^{10} f(x) \, dx &= 3 \\
\int_1^5 g(x) \, dx &= 16 & \int_5^2 g(x) \, dx &= 10
\end{align*}
\]

(a) What is \( \int_0^{10} f(x) \, dx \)?

By the additivity property of definite integrals, we have that

\[
\int_0^{10} f(x) \, dx = \int_0^7 f(x) \, dx + \int_7^{10} f(x) \, dx
\]

We know both quantities on the right-hand side, so we can just add them together to get that

\[
\int_0^{10} f(x) \, dx = 9 + 3 = 12
\]

(b) What is \( \int_1^2 g(x) \, dx \)?

By the additivity property of definite integrals, we have that

\[
\int_1^5 g(x) \, dx = \int_1^2 g(x) \, dx + \int_2^5 g(x) \, dx
\]

We know the quantity on the left-hand side and one of the quantities on the right-hand side, so we get that

\[
16 = \int_1^2 g(x) \, dx + 10
\]

Subtracting, we get that \( \int_1^2 g(x) \, dx = 6 \).