1. (8 points)

(a) Find the average rate of change of $f(x) = \sqrt{x}$ from $x = 4$ to $x = 16$.

**Solution.**

\[
\frac{f(16) - f(4)}{16 - 4} = \frac{\sqrt{16} - \sqrt{4}}{16 - 4} = \frac{4 - 2}{16 - 4} = \frac{2}{12} = \frac{1}{6}
\]

(b) Find the average rate of change of $f(x) = \sin(x)$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

**Solution.**

\[
\frac{f\left(\frac{\pi}{2}\right) - f\left(-\frac{\pi}{2}\right)}{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)} = \frac{\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)}{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)} = \frac{1 - (-1)}{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)} = \frac{2}{\pi}
\]
2. (4 points) Below is the graph of a function $f(x)$. Draw the secant line between $(0, f(0))$ and $(2, f(2))$, and then find the slope of this secant line.

Solution.

To find the slope of the secant line, use the points $(0, 1)$ and $(2, 4)$ in the slope formula:

$$\frac{4 - 1}{2 - 0} = \frac{3}{2}$$
3. (8 points) Let \( f(x) = x^2 + 3x + 1 \).

(a) Find the slope of the secant line between (0, \( f(0) \)) and (1, \( f(1) \)).

**Solution.** First calculate \( f(0) = 1 \) and \( f(1) = 5 \). Then use the slope formula:

\[
\frac{5 - 1}{1 - 0} = 4
\]

(b) Find the slope of the secant line between (0, \( f(0) \)) and \((h, f(h))\).

**Solution.** Again, \( f(0) = 1 \), so \((0, f(0)) = (0, 1)\). Use the slope formula, using \( h \) and \( f(h) \):

\[
\frac{f(h) - 1}{h - 0} = \frac{h^2 + 3h + 1 - 1}{h} = \frac{h^2 + 3h}{h} = \frac{h(h + 3)}{h} = h + 3
\]

(c) Use your answer to part (b) to estimate the slope of the tangent line to \( f(x) \) at \( x = 0 \).

**Solution.** Secant lines where \( h \) is small will have slopes very close to the slope of the tangent line. Notice that as \( h \) gets small, the quantity \( h + 3 \) (the slope of the secant line from part (b)) gets close to 3. So the slope of the tangent line is 3.

(d) Come up with an equation for the tangent line at \( x = 0 \).

**Solution.** We determined in part (c) that the tangent line has slope 3. Also, it goes through the point \((0, f(0)) = (0, 1)\) on the graph of \( f \). So use the point-slope formula for a line:

\[
y - 1 = 3(x - 0) \\
y = 3x + 1
\]