1. \( f(x) = x^2 + 2x + 1 \)

**Solution:** Just use the power rule: \( f'(x) = 2x + 2 \)

2. \( g(x) = 3xe^x \)

**Solution:**
\[
\begin{align*}
g'(x) &= (3xe^x)' \\
&= 3(xe^x)' \\
&= 3((x)'e^x + x(e^x)') \\
&= 3(e^x + xe^x) \\
&= 3e^x(1 + x)
\end{align*}
\]

3. \( h(x) = \ln(x^2 + x) \)

**Solution:** Use the chain rule with the outer function \( f(x) = \ln(x) \) and inner function \( g(x) = x^2 + x \). Then \( h(x) = \ln(x^2 + x) = f(g(x)) \), and the chain rule says
\[
h'(x) = f'(g(x)) \cdot g'(x) = \frac{1}{x^2 + x} \cdot (2x + 1) = \frac{2x + 1}{x^2 + x}
\]

4. \( 4(x) = 3x^3e^7 = 3e^7x^3 \)

**Solution:** Since \( 3e^7 \) is just a constant, use the power rule: \( t'(x) = 9e^7x^2 \).

5. \( n(x) = \frac{x + 1}{x - 1} \)

**Solution:**
\[
n'(x) = \frac{(x + 1)'(x - 1) - (x + 1)(x - 1)'}{(x - 1)^2} \quad \text{(quotient rule)}
\]
\[
= \frac{1 \cdot (x - 1) - (x + 1) \cdot 1}{(x - 1)^2}
\]
\[
= \frac{-2}{(x - 1)^2}
\]
6. \( a(t) = te^{t^2} \)

**Solution:** First, by the product rule, we have

\[
a'(t) = (t)e^{t^2} + t(e^{t^2})' = e^{t^2} + t(e^{t^2})'
\]

So we just need to compute \((e^{t^2})'\), the derivative of \(e^{t^2}\). To do so, use the chain rule, with outer function \(f(t) = e^t\) and inner function \(g(t) = t^2\). Then \(e^{t^2} = f(g(t))\), and so

\[
(e^{t^2})' = f'(g(t)) \cdot g'(t) = e^{t^2} \cdot 2t
\]

So finally we have

\[
a'(t) = e^{t^2} + t(e^{t^2})' = e^{t^2} + 2t^2 e^{t^2}
\]

7. \( f(u) = \frac{u^2}{\ln(1 + e^u)} \)

**Solution:** First, use the quotient rule to get

\[
f'(u) = \frac{(u^2)' \ln(1 + e^u) - u^2 (\ln(1 + e^u))'}{(\ln(1 + e^u))^2}
\]

\[
= \frac{2u \ln(1 + e^u) - u^2 (\ln(1 + e^u))'}{(\ln(1 + e^u))^2}
\]

To evaluate the derivative of \(\ln(1 + e^u)\), use the chain rule with outer function \(F(u) = \ln(u)\) and \(G(u) = 1 + e^u\) to get

\[
(\ln(1 + e^u))' = F'(G(u)) \cdot G'(u) = \frac{1}{1 + e^u} \cdot e^u
\]

So finally we have

\[
f'(u) = \frac{2u \ln(1 + e^u) - u^2 e^u}{(\ln(1 + e^u))^2}
\]
8. \( g(x) = e^{\sqrt[4]{3x^4 + 3x^2 + 1}} \)

**Solution:** Use chain rule with outer function \( f(x) = e^x \) and inner function \( h(x) = \sqrt[4]{3x^4 + 3x^2 + 1} \):

\[
g'(x) = f'(h(x)) \cdot h'(x) = e^{\sqrt[4]{3x^4 + 3x^2 + 1}} \cdot h'(x)
\]

To compute \( h'(x) \), use the chain rule again, with outer function \( F(x) = \sqrt[4]{x} = x^{1/4} \) and inner function \( G(x) = 3x^4 + 3x^2 + 1 \):

\[
h'(x) = F'(G(x)) \cdot G'(x) = \frac{1}{4} (3x^4 + 3x^2 + 1)^{-3/4} (12x^3 + 6x)
\]

So, all together, we have

\[
g'(x) = e^{\sqrt[4]{3x^4 + 3x^2 + 1}} \cdot h'(x) = \frac{1}{4} e^{\sqrt[4]{3x^4 + 3x^2 + 1}} (3x^4 + 3x^2 + 1)^{-3/4} (12x^3 + 6x)
\]

9. \( h(y) = \frac{1}{(7y)^2} \)

**Solution:** First, square the bottom to make things simpler:

\[
h(y) = \frac{1}{7^2y^2} = \frac{1}{49y^2} = \frac{1}{49} y^{-2}
\]

Now just use the power rule:

\[
h'(y) = -\frac{2}{49} y^{-3} = -\frac{2}{49y^3}
\]
10. \( s(x) = \frac{(5x^3 + 2x^2 + 2) \ln(x)}{e^{3x} + x} \)

**Solution:** Let \( f(x) = (5x^3 + 2x^2 + 2) \ln(x) \) be the numerator. Then the quotient rule tells us that

\[
s'(x) = \frac{f'(x)(e^{3x} + x) - f(x)(3e^{3x} + 1)}{(e^{3x} + x)^2}
\]

To compute \( f'(x) \), use the product rule:

\[
f'(x) = (15x^2 + 4x) \ln(x) + \frac{5x^2 + 2x^2 + 2}{x}
\]

Finally, plug this back into our formula for \( s'(x) \):

\[
s'(x) = \left( (15x^2 + 4x) \ln(x) + \frac{5x^2 + 2x^2 + 2}{x} \right) \left( e^{3x} + x \right) - (5x^3 + 2x^2 + 2)(\ln(x)(3e^{3x} + 1))
\]

11. \( f(r) = (r^2 + r)^{100} \)

**Solution:** Just use the chain rule with outer function \( g(r) = r^{100} \) and inner function \( h(r) = r^2 + r \):

\[
f'(r) = g'(h(r)) \cdot h'(r) = 100(r^2 + r)^{99}(2r + 1)
\]

12. \( g(p) = (3p^2 + p + 1)e^p \ln(p) \)

**Solution:** From the product rule, we have

\[
g'(p) = (6p + 1)e^p \ln(p) + (3p^2 + p + 1)e^p \ln(p) + (3p^2 + p + 1)e^p p
\]

13. \( h(x) = \frac{(x^2 + x + 1)(4^x)}{x \ln(x)} \)

**Solution:** Since this just a fraction, we use the quotient rule:

\[
h'(x) = \frac{[(x^2 + x + 1)(4^x)]' (x \ln(x)) - (x^2 + x + 1)(4^x)(x \ln(x))'}{(x \ln(x))^2}
\]
Now we just need to compute the two derivatives \([(x^2 + x + 1)(4^x)]'\) and \((x \ln(x))'\). Use the product rule for both, and we get

\[
[(x^2 + x + 1)(4^x)]' = (x^2 + x + 1)'(4^x) + (x^2 + x + 1)(4^x)' = (2x + 1)4^x + (x^2 + x + 1)\ln(4)4^x
\]

and

\[
(x \ln(x))' = x'(\ln(x)) + x(\ln(x))' = \ln(x) + \frac{x}{x} = \ln(x) + 1
\]

Now plug these back into our original equation:

\[
h'(x) = \frac{[(2x + 1)4^x + (x^2 + x + 1)\ln(4)4^x](x \ln(x)) - (x^2 + x + 1)(4^x)(\ln(x) + 1)}{(x \ln(x))^2}
\]

14. \(t(x) = \ln(x^2 + 3x)e^{x^2-x}\)

**Solution:** The function is the product of the two functions \(\ln(x^2 + 3x)\) and \(e^{x^2-x}\), so use the product rule:

\[
t'(x) = (\ln(x^2 + 3x))'e^{x^2-x} + \ln(x^2 + 3x)(e^{x^2-x})'
\]

To compute the derivative of \(\ln(x^2 + 3x)\), use the chain rule, where the outer function is \(\ln(x)\) and the inner function is \(x^2 + 3x\) to get

\[
(\ln(x^2 + 3x))' = \frac{2x + 3}{x^2 + 3x}
\]

To compute the derivative of \(e^{x^2-x}\), also use the chain rule, where the outer function is \(e^x\) and the inner function is \(x^2 - x\), to get

\[
(e^{x^2-x})' = e^{x^2-x}(2x - 1)
\]

Now put it all together to get

\[
t'(x) = \frac{2x + 3}{x^2 + 3x}e^{x^2-x} + \ln(x^2 + 3x)e^{x^2-x}(2x - 1)
\]

15. \(n(x) = \frac{1}{\ln(x) + x}\)

**Solution:** There are two ways to do this problem:

(a) Use quotient rule

(b) Use the chain rule where the outer function is \(\frac{1}{x}\) and the inner function is \(\ln(x) + x\).

We will do it both ways. First:

(a) Use the quotient rule:

\[
n'(x) = \frac{(\ln(x) + x)' - 1 \cdot (\ln(x) + x)'}{(\ln(x) + x)^2}
\]
Since the derivative of the constant function 1 is zero, this gives

\[ n'(x) = -\left(\frac{1}{x} + 1\right) \]

The alternative way is:

(b) Use the chain rule. The outer function is \( \frac{1}{x} \) and the inner function is \( \ln(x) + x \). Since the derivative of \( \frac{1}{x} \) is \( -\frac{1}{x^2} \), this gives

\[ n'(x) = \frac{-1}{(\ln(x) + x)^2} \cdot \left(\frac{1}{x} + 1\right) \]

16. \( q(p) = p(p^3 + p + e^p) \)

**Solution:** We could use the product rule right away, but let’s multiply it out first:

\[ q(p) = p^4 + p^2 + pe^p \]

We still have to use the product rule on the last term, but now we can just use the power rule for the first two terms:

\[ q'(p) = 4p^3 + 2p + (pe^p)' = 4p^3 + 2p + e^p + pe^p \]

17. \( f(u) = \frac{e^7 + \ln(2) + 1}{u} - 1 \)

**Solution:** Write \( \frac{1}{u} \) as \( u^{-1} \):

\[ f(u) = (e^7 + \ln(2) + 1)u^{-1} - 1 \]

The derivative of the constant -1 is zero, and just use the power rule for the first term

\[ f'(u) = -(e^7 + \ln(2) + 1)u^{-2} = \frac{- (e^7 + \ln(2) + 1)}{u^2} \]

18. \( g(x) = \sqrt{x^3 + 2x + 1} \)

**Solution:** Just use the chain rule with outer function \( \sqrt{x} \) and inner function \( x^3 + 2x + 1 \):

\[ g'(x) = \frac{3x^2 + 2}{2\sqrt{x^3 + 2x + 1}} \]

19. \( h(y) = e^{5y} + \ln(2y) + \frac{1}{y^3} \)
**Solution:** For the derivative of $e^{5y}$ use the chain rule with outer function $e^y$ and inner function $5y$ to get

$$ (e^{5y})' = 5e^{5y} $$

For the derivative of $\ln(2y)$, use the chain rule with outer function $\ln(y)$ and inner function $2y$:

$$ (\ln(2y))' = \frac{2}{2y} = \frac{1}{y} $$

Put it together (using just the power rule on the last term) to get

$$ h'(y) = 5e^{5y} + \frac{1}{y} - \frac{5}{y^6} $$

20. $s(t) = \frac{t^3 + 7t + 1}{t}$

**Solution:** Use the quotient rule:

$$ s'(t) = \frac{(t^3 + 7t + 1)'t - (t^3 + 7t + 1)(t)'}{t^2} = \frac{(3t^2 + 7)t - (t^3 + 7t + 1)}{t^2} $$

21. $f(x) = \ln\left(e^{x^2 + 1}\right)$

**Solution:** We have to use the chain rule with outer function $F(x) = \ln(x)$ and inner function $G(x) = e^{x^2 + 1}$. So

$$ f'(x) = F'(G(x)) \cdot G'(x) $$

We know that $F'(x) = \frac{1}{x}$, but for $G'(x)$, we must use the product rule:

$$ G'(x) = (e^x)'(x^2 + 1) + e^x(x^2 + 1)' 
= e^x(x^2 + 1) + e^x(2x) 
= e^x(x^2 + 2x + 1) 
= e^x(x + 1)^2 $$

Now put it all together:

$$ f'(x) = \frac{e^x(x + 1)^2}{e^x(x^2 + 1)} = \frac{(x + 1)^2}{x^2 + 1} $$

22. $g(x) = e^{(x^2+3)^2(x-1)^5}$

**Solution:** We must of course use the chain rule, with outer function $e^x$ and inner function $(x^2 + \ldots$
3) $7(x - 1)^5$. The derivative of the inner function (using both product and chain rules) is

$$[(x^2 + 3)^7(x - 1)^5]' = 7(x^2 + 3)^6(2x)(x - 1)^5 + (x^2 + 3)^7 \cdot 5(x - 1)^4$$

$$= (x^2 + 3)^6(x - 1)^4(14x(x - 1) + 5(x^2 + 3))$$

$$= (x^2 + 3)^6(x - 1)^4(19x^2 - 14x + 15)$$

Use what we just found plus the chain rule (with the original function) to get

$$g'(x) = e^{(x^2+3)^7(x-1)^5} \cdot (x^2 + 3)^6(x - 1)^4(19x^2 - 14x + 15)$$

23. $h(x) = \ln(x) - \ln\left(\frac{1}{x}\right)$

**Solution:** For the derivative of $\ln\left(\frac{1}{x}\right)$, use the chain rule:

$$\left(\ln\left(\frac{1}{x}\right)\right)' = \frac{1}{x} \cdot \frac{-1}{x^2}$$

$$= \frac{-x}{x^2}$$

$$= \frac{-1}{x}$$

So the derivative of the original function is

$$h'(x) = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$$

24. $t(x) = x(2^x - \sqrt{5})$

**Solution:** Use the product rule

$$t'(x) = (x)'(2^x - \sqrt{5}) + x(2^x - \sqrt{5})' = x^2 - \sqrt{5} + x(2)$$

25. $n(x) = (x + 1)^4(x - 1)^4x^3$

**Solution:** Use the product rule, and then the chain rule:

$$n'(x) = [(x + 1)^4]'(x - 1)^4x^3 + (x + 1)^4[(x - 1)^4]'x^3 + (x + 1)^4(x - 1)^4[x^3]'$$

$$= 4(x + 1)^3(x - 1)^4x^3 + 4(x + 1)^4(x - 1)^3x^3 + 3(x + 1)^4(x - 1)^4x^2$$

26. $a(t) = \frac{t^2 - 1}{e^{2t} - 1}$
Solution: Use the quotient rule:

\[ a'(t) = \frac{(t^2 - 1)'(e^{2t} - 1) - (t^2 - 1)(e^{2t} - 1)'}{(e^{2t} - 1)^2} = \frac{2t(e^{2t} - 1) - (t^2 - 1)2e^{2t}}{(e^{2t} - 1)^2} \]

27. \( f(u) = \frac{u^4}{\ln(1 + e^u)} \)

Solution: Use the quotient rule (and the chain rule for the derivative of the denominator):

\[ f'(u) = \frac{(u^4)'\ln(1 + e^u) - u^4(\ln(1 + e^u))'}{(\ln(1 + e^u))^2} = \frac{4u^3\ln(1 + e^u) - u^4e^u}{(\ln(1 + e^u))^2} \]

28. \( g(x) = \sqrt{\frac{x^2 + x}{\ln(x) + 1}} \)

Solution: Use the chain rule with outer function \( F(x) = \sqrt{x} \) and inner function \( G(x) = \frac{x^2 + x}{\ln(x) + 1} \):

\[ g'(x) = F'(G(x)) \cdot G'(x) = \frac{G'(x)}{2\sqrt{\frac{x^2 + x}{\ln(x) + 1}}} \]

To compute \( G'(x) \), use the quotient rule:

\[ G'(x) = \frac{(2x + 1)(\ln(x) + 1) - (x^2 + x) \cdot \frac{1}{x}}{(\ln(x) + 1)^2} \]

Plug this in for \( G'(x) \) in the first formula to get the final answer.

29. \( s(t) = \frac{1}{|t|} \)

Solution: Since \( |t| = t \) if \( t \) is positive and \( |t| = -t \) if \( t \) is negative, we can think of the function as a piecewise function:

\[ s(t) = \begin{cases} \frac{1}{t} & \text{for } t > 0 \\ -\frac{1}{t} & \text{for } t < 0 \end{cases} \]

To take the derivative, just differentiate the “pieces”:

\[ s'(t) = \begin{cases} -\frac{1}{t^2} & \text{for } t > 0 \\ \frac{1}{t^2} & \text{for } t < 0 \end{cases} \]
30. $h(y) = |y^2|$

**Solution:** Since $y^2 \geq 0$ for any value of $y$, the absolute value bars are unnecessary, and

$$h(y) = y^2$$

So the derivative is just

$$h'(y) = 2y$$