MATHEMATICAL OLYMPIAD 2004
7–9 grades

Calculators are prohibited!

1. Two players play the following game. On the lowest left square of an $8 \times 8$ chessboard there is a rook. The first player is allowed to move the rook up or to the right by an arbitrary number of squares. The second player is also allowed to move the rook up or to the right by an arbitrary number of squares. Then the first player is allowed to do this again, and so on. The one who moves the rook to the upper right square wins. Who has a winning strategy?

2. In Crocodile Country there are banknotes of 1 dollar, 10 dollars, 100 dollars, and 1,000 dollars. Is it possible to get 1,000,000 dollars by using 250,000 banknotes?

3. Fifteen positive numbers (not necessarily whole numbers) are placed around the circle. It is known that the sum of every four consecutive numbers is 30. Prove that each number is less than 15.

4. Donald Duck has 100 sticks, each of which has length 1 cm or 3 cm. Prove that he can break into 2 pieces no more than one stick, after which he can compose a rectangle using all sticks.

5. Three consecutive 2 digit numbers are written next to each other. It turns out that the resulting 6 digit number is divisible by 17. Find all such numbers.