1. A triangle ABC is drawn in the plane. Chose a point D inside the triangle. Show that the sum of distances AD+BD+CD is less than the perimeter of the triangle.

**Solution.** Let us consider the case when D belongs to the segment AB. Construct parallelogram CASB by drawing side BS parallel to CA, and side AS parallel to CB. Extend ray CD to the intersection with BS. Let K be the intersection point of CK and BS (see Fig).

Note that $CD \leq CK \leq CB + BK \leq CB + BS = CB + AC$. Hence, $AD + BD + CD \leq AB + AC + BC$, or, equivalently, $CD \leq AC + BC$.

Let now D is a point inside the triangle ABC. Extend segment CD to CK where K is the intersection of ray CD with side AB (see Fig., below). Draw segment LM parallel to the base AB, segment DS parallel to side AC and segment DK parallel side BC (see Fig.)

Then, as we discussed above, $CD \leq CL + CM$. By triangle inequality, $AD \leq AL + LD = AL + AS$, and $BD \leq BM + MD = BM + BK$. Adding all inequalities we obtain,

$$AD + BD + CD \leq AC + BC + AS + BK \leq AB + BC + AC.$$
2. In a triangle ABC the bisector of the angle C intersects the side AB at M, and the bisector of the angle A intersects CM at the point T. Suppose that the segments CM and AT divided the triangle ABC into three isosceles triangles. Find the angles of the triangle ABC.

Solution. From the Fig.

\begin{center}
\includegraphics[width=0.5\textwidth]{triangle.png}
\end{center}

we conclude that $5\alpha = 180$. Hence, $\alpha = 36$, therefore angles are $36^\circ$ and $72^\circ$.

3. You are given 100 weights of masses 1, 2, 3, \ldots, 99, 100. Can one distribute them into 10 piles having the following property: the heavier the pile, the fewer weights it contains?

Solution. Answer: impossible.

The sum of all masses is 5050. The heaviest pile must weight more than 505, and hence contains more than 6 weights. Therefore the next pile contains at least 7 weights, the next at least 8 weights, and so on and so forth. We obtain that the number of weights is at least $6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = 105$ which contradicts to the condition that there is only 100 weights.
4. Each cell of a $10 \times 10$ table contains a number. In each row the greatest number (or one of the largest, if more than one) is underscored, and in each column the smallest (or one of the smallest) is also underscored. It turned out that all of the underscored numbers are underscored exactly twice. Prove that all numbers stored in the table are equal to each other.

**Solution.** Let $A$ be the the greatest number. If $A$ is underscored twice then all integers in the column containing $A$ are equal to $A$. Let another column contains the least integer $B$ such that $B < A$. Then $B$ is underscored only once. Hence, all numbers are equal to each other.  

5. Two stores have warehouses in which wheat is stored. There are 16 more tons of wheat in the first warehouse than in the second. Every night exactly at midnight the owner of each store steals from his rival, taking a quarter of the wheat in his rival’s warehouse and dragging it to his own. After 10 days, the thieves are caught. Which warehouse has more wheat at this point and by how much?

**Solution.** Let $x + \delta, x$ be amounts of wheat in the first and the second warehouse. Then the next day amounts of wheat are $3/4x + 3/4\delta + 1/4x = x + 3/4\delta$ and $3/4x + 1/4x + 1/4\delta = x + 1/4\delta$. The difference becomes $3/4\delta - 1/4\delta = \delta/2$. Each day the difference becomes twice smaller. Therefore after 10 days the difference becomes $16 \cdot 2^{-10} = 2^{-6} = 1/64$ ton.