1. We say that integers $a$ and $b$ are friends if their product is a perfect square. Prove that if $a$ is a friend of $b$, then $a$ is a friend of $\gcd(a,b)$. (Recall that $\gcd(a,b)$ is the greatest common divisor of $a$ and $b$.)

2. On the island of knights and liars, a traveler visited his friend, a knight, and saw him sitting at a round table with five guests.
   “I wonder how many knights are among you?” he asked.
   “Ask everyone a question and find out yourself” advised him one of the guests.
   “Okay. Tell me one: Who are your neighbors?” asked the traveler.
   This question was answered the same way by all the guests.
   “This information is not enough!” said the traveler.
   “But today is my birthday, do not forget it!” said one of the guests.
   “Yes, today is his birthday!” said his neighbor. Now the traveler was able to find out how many knights were at the table.
   Indeed, how many of them were there if knights always tell the truth and liars always lie?

3. A rope is folded in half, then in half again, then in half yet again. Then all the layers of the rope were cut in the same place. What is the length of the rope if you know that one of the pieces obtained has length of 9 meters and another has length 4 meters?
4. The floor plan of the palace of the Shah is a square of dimensions $6 \times 6$, divided into rooms of dimensions $1 \times 1$. In the middle of each wall between rooms is a door. The Shah orders his architect to eliminate some of the walls so that all rooms have dimensions $2 \times 1$, no new doors are created, and a path between any two rooms has no more than $N$ doors. What is the smallest value of $N$ such that the order could be executed?

5. There are 10 consecutive positive integers written on a blackboard. One number is erased. The sum of remaining nine integers is 2011. Which number was erased?