MATHEMATICS OLYMPIAD 2012
Grades 5–6

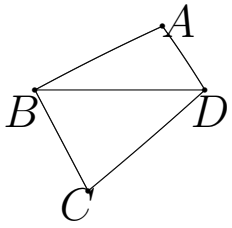
1. A boy has as many sisters as brothers. However, his sister has twice as many brothers as sisters. How many boys and girls are there in the family?

2. Solve each of the following problems.
   (1) Find a pair of numbers with a sum of 11 and a product of 24.
   (2) Find a pair of numbers with a sum of 40 and a product of 400.
   (3) Find three consecutive numbers with a sum of 333.
   (4) Find two consecutive numbers with a product of 182.

3. 2008 integers are written on a piece of paper. It is known that the sum of any 100 numbers is positive. Show that the sum of all numbers is positive.

4. Let p and q be prime numbers greater than 3. Prove that $p^2 - q^2$ is divisible by 24.

5. Four villages A,B,C, and D are connected by trails as shown on the map.



On each path $A \to B \to C$ and $B \to C \to D$ there are 10 hills, on the path $A \to B \to D$ there are 22 hills, on the path $A \to D \to B$ there are 45 hills.

A group of tourists starts from A and wants to reach D. They choose the path with the minimal number of hills. What is the best path for them?