Homework:

1. A box contains 300 matches. Players take turns removing no more than half of the matches in the box. The player who cannot move loses. Who will win, and how?

2. In an $8 \times 8$ table one of the boxes is colored black and all the others are white. Prove that one can not make all the boxes white by recoloring the rows and the columns. "Recoloring" is the operation of changing the color of all the boxes in a row or in a column.

3. Solve the same problem for a $3 \times 3$ table if initially there is only one black box in a corner of the table.

4. There are six sparrows sitting on six trees, one sparrow on each tree. The trees stand in a row, with 10 meters between any neighboring trees. If a sparrow flies from some tree to another, then at the same time other sparrow flies from some tree to another the same distance away, but in the opposite direction. Is it possible for all sparrows to gather on one tree? What if there are seven sparrows and seven trees?