Let $L$ be a standard Lie algebra with Chevalley basis $\mathcal{C} = (x_\alpha, h_\beta \mid \alpha \in \Phi, \beta \in \Pi)$.

1. Let $I$ be a finite set and for $i \in I$, let $V_i$ be an $L$-module. Let $\Psi \subseteq \Phi$, $m \in \mathbb{N}^\Psi$ and $v_i \in V_i$. Then

$$\frac{x^m}{m!} \bigotimes_{i \in I} v_i = \sum \left\{ \bigotimes_{i \in I} \frac{x^{m_i}}{m_i!} \cdot v_i \mid m_i \in \mathbb{N}^\Psi, \sum_{i \in I} m_i = m. \right\}$$

2. Let $\lambda$ be a minimal dominant integral weight for $\hat{\Phi}$. Show that $W(\Phi)$ acts transitively on the set of weights for $H$ on $V(\lambda)$.

3. Suppose $\Phi$ is of type $B_n$ and let $\alpha$ be the unique long root in $\Pi$. Determine $\dim V(\alpha^*)$.

4. Suppose $\Phi$ is of type $G_2$. For all $\alpha, \beta \in \Phi$ with $\alpha + \beta \in \Phi$ compute $k_{\alpha \beta}$. (Note that sign of $k_{\alpha \beta}$ depends on the choice of $\mathcal{C}$, make choices such that the $k_{\alpha \beta}$ become unique).