1. Let $K$ be a standard field, $L = \mathfrak{sl}(K^n)$ and let $H$ be the subalgebra consisting of the diagonal matrices in $L$. Put $E_i = E_{ii}$ (and so $H = \{ \sum_{i=1}^n k_i E_i \mid k_i \in K, \sum_{i=1}^n k_i = 0 \}$). Let $f = f_L, f^*, \Lambda, \Phi, t_\alpha, h_\alpha$ and $\omega_\alpha$ be defined as in class. Define $\lambda_j : H \to K, \sum k_i E_i \to k_j$. Let $\alpha \in \Phi$.

(a) Show that $KE_{ij}$ is an $H$-submodule of $L$ with weight $\lambda_i - \lambda_j$.
(b) Determine $\Lambda$ and $L_\alpha$.
(c) Show that $H$ is a Cartan subalgebra of $L$.
(d) Compute $f(\sum k_i E_i, \sum l_i E_i)$.
(e) For all $1 \leq i, j \leq n$ compute $f^*(\lambda_i, \lambda_j)$.
(f) Determine $t_\alpha$ and $h_\alpha$
(g) For $1 \leq i \leq n$ compute $\omega_\alpha(\lambda_i)$.
(h) Determine all the $\alpha$-strings in $\Lambda$.

2. Let $L$ be simple and finite dimensional and suppose that $K$ is algebraically closed. Let $f$ and $g$ be $L$-invariant bilinear forms on $L$ with $f \neq 0$. Show that there exists $k \in K$ with $g = kf$. (Hint: Use the corresponding maps $\tilde{f}, \tilde{g} : L \to L^*$ and Schur’s Lemma)