Quiz 9/Solutions

1. Starting with $x_1 = 0$ use one iteration of Newton's method to approximate a solution of the equation $x^4 - 2x + 1 = 0$.

$$f(x) = x^4 - 2x + 1$$
$$f'(x) = 4x^3 - 2$$

Solution 1:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0 - \frac{0^4 - 2\cdot 0 + 1}{4\cdot 0^3 - 2} = 0 - \frac{1}{-2} = \boxed{\frac{1}{2}}.$$

□

Solution 2:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 - 2x_n + 1}{4x_n^3 - 2} = \frac{(4x_n^4 - 2x_n) - (x_n^4 - 2x_n + 1)}{4x_n^3 - 2} = \frac{3x_n^4 - 1}{4x_n^3 - 2}$$

and so

$$x_2 = \frac{3\cdot 0^4 - 1}{4\cdot 0^3 - 2} = \boxed{\frac{1}{2}}.$$

□

Solution 2 is longer than Solution 1, but would be shorter if one is required to carry out several iterations of Newton's Method.

2. Let f be a differential function. Determine f if $f'(x) = 3x^2 + \sin x$ and $f(0) = -2$.

Note that f is an antiderivative of $f'(x) = 3x^2 + \sin x$. So

$$f(x) = x^3 - \cos x + C,$$

C a constant. Since $f(0) = -2$ we get

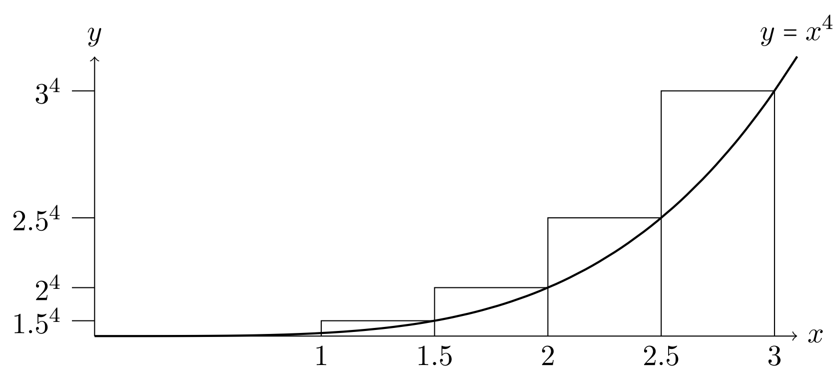
$$-2 = f(0) = 0^3 - \cos 0 + C = -1 + C, \quad \text{so} \quad C = -2 + 1 = -1.$$

Hence

$$\boxed{f(x) = x^3 - \cos x - 1}.$$

□

3. Determine the approximation sum R_4 for the area of the region under the curve $y = x^4$ from $x = 1$ to $x = 3$. **Do not evaluate the sum and do not simplify.**



To compute R_4 we need to divide the interval $[1,3]$ into four intervals of equal length

$$\Delta x = \frac{3-1}{4} = 0.5:$$

$$x_0 = 1, \quad x_1 = 1.5, \quad x_2 = 2, \quad x_3 = 2.5, \quad x_4 = 3$$

and determine the right endpoints of each interval

$$1.5, \quad 2, \quad 2.5, \quad 3$$

So

$$\begin{aligned} R_4 &= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x \\ &= 1.5^4 \cdot 0.5 + 2^4 \cdot 0.5 + 2.5^4 \cdot 0.5 + 3^4 \cdot 0.5 \end{aligned}$$