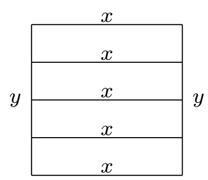
Quiz 8/Solutions

A 180 m^2 rectangular pea patch is to be enclosed by a fence and divided into four equal parts by three additional fences parallel to one of the sides. What dimensions for the outer fence will require the smallest amount of total length of fence? How much fence will be needed?



We need to minimize the total length L of the fence. From the diagram we see that

$$L = 5x + 2y.$$

The area of outside rectangle has to be 180. So

$$xy = 180.$$

Note that $x \geq 0$ and $y \geq 0$. But as $xy = 180$, x cannot be zero. Thus the domain for x is $(0, \infty)$. From $xy = 180$ we get

$$y = \frac{180}{x}$$

and so

$$L = 5x + 2y = 5x + 2\frac{180}{x} = 5x + \frac{360}{x} = 5(x + \frac{72}{x}).$$

Thus

$$L' = 5(1 - \frac{72}{x^2}) = 5\frac{x^2 - 72}{x^2}.$$

Hence L' is defined for all $x > 0$ and $L' = 0$ when $x^2 = 72$. Since $x > 0$, we see that $L' = 0$ for $x = \sqrt{72}$. To determine where L is increasing and decreasing:

	$(0, \sqrt{72})$	$(\sqrt{72}, \infty)$
$x^2 - 72$	$-$	$+$
x^2	$+$	$+$
L'	$-$	$+$
L	$\searrow$	$\nearrow$

Hence the First Derivative Test for Extreme Values shows that L has an absolute minimum at $x = \sqrt{72}$. Note that $\sqrt{72} = \sqrt{6^2 \cdot 2} = 6\sqrt{2}$. So for $x = \sqrt{72}$:

$$y = \frac{180}{x} = \frac{180}{6\sqrt{2}} = \frac{6 \cdot 15 \cdot 2}{6\sqrt{2}} = 15\sqrt{2} \quad \text{and} \quad L = 5x + 2y = 5 \cdot 6 \cdot \sqrt{2} + 2 \cdot 15 \cdot \sqrt{2} = 30\sqrt{2} + 30\sqrt{2} = 60\sqrt{2}.$$

So the dimensions of the outer fence are

$$\boxed{6\sqrt{2}\,\mathrm{m} \times 15\sqrt{2}\,\mathrm{m}} \quad \left(\text{which is equal to } \boxed{\sqrt{72}\,\mathrm{m} \times \sqrt{450}\,\mathrm{m}}\right)$$

Also

$$\boxed{60\sqrt{2}\,\mathrm{m}} \quad \left(\text{which is equal to } \boxed{\sqrt{720}\,\mathrm{m}}\right)$$

of fence are needed.