MTH 132-020 **Calculus I** **F18**

Quiz 6/Solutions

1. Compute the linearization $L(x)$ of the function $f(x) = \frac{1}{(x-1)^3}$ at 0.

Recall that the linearization of $f(x)$ at a is

$$L(x) = f(a) + f'(a)(x-a).$$

Since $a = 0$ we compute:

$$\begin{aligned}
f(0) &= \frac{1}{(0-1)^3} = \frac{1}{-1}^{3} = \frac{1}{-1} = -1 \\
f'(x) &= \left(\frac{1}{(x-1)^3}\right)' = -3\frac{1}{(x-1)^4} \\
f'(1) &= -3\frac{1}{(0-1)^4} = -\frac{3}{(-1)^4} = -\frac{3}{1} = -3
\end{aligned}$$

Thus

$$L(x) = f(0) + f'(0)(x-0) = -1 + (-3)x = \boxed{-1-3x}.$$

□

2. Suppose $f(2) = 5$ and $f'(2) = 3$. Use linear approximation to estimate $f(2.1)$.

I will present two solutions:
Solution 1:
The linerarization $L(x)$ of f at 2 is:

$$L(x) = f(2) + f'(2)(x-2) = 5 + 3(x-2).$$

and so

$$f(2.1) \approx L(2.1) = 5 + 3(2.1-2) = 5 + 3\cdot 0.1 = 5 + 0.3 = \boxed{5.3}.$$

Solution 2:

$$\begin{aligned}
\mathrm{d}x &= \Delta x = 2.1 - 2 = 0.1 \\
\mathrm{d}y &= f'(1)\mathrm{d}x = 3\cdot 0.1 = 0.3 \\
f(2.1) &= f(2) + \Delta y \approx f(2) + \mathrm{d}y = 5 + 0.3 = \boxed{5.3}
\end{aligned}$$

□

3. Find the absolute maximum value and the absolute minimum value of $f(x) = 12x - x^3$ on the interval $[-1,3]$.

The function $f(x) = 12x - x^3$ is continuous on the closed interval $[-1,3]$. So we can use the "Continuous closed interval" method to compute the absolute extrema of f on $[-1,3]$.

I) Find the critical numbers of f in $(-1,3)$:

$$f'(x) = (12x - x^3)' = 12 - 3x^2 = 3(4-x^2) = 3(x-2)(x+2)$$

Observe that $f'(x)$ exist for all x in $(-1,3)$. Also $f'(x) = 0$ for $x = -2$ and $x = 2$. But -2 is not in $(-1,3)$ so $x = 2$ is the only critical number of f in $(-1,3)$.

II) Compute the values of f at the endpoints and the critical points:

$$
\begin{aligned}
f(-1) &= 12 \cdot -1 - (-1)^3 = -12 - (-1) = -11 \\
f(2) &= 12 \cdot 2 - 2^3 \qquad = 24 - 8 \qquad = 16 \\
f(3) &= 12 \cdot 3 - 3^3 \qquad = 36 - 27 \qquad = 9
\end{aligned}
$$

III) The largest value in II) is 16, so the absolute maximum value of $f(x)$ on $[-1, 3]$ is $\boxed{16}$ attained at $x = 2$.

IV) The smallest value in II) is -11, so the absolute minimum value of $f(x)$ on $[-1, 3]$ is $\boxed{-11}$ attained at $x = -1$. □

4. Fill in the blanks:

The Mean Value Theorem Let f be a function defined on the closed interval $[a, b]$. Suppose that

1. f is ___continuous___ on the ___closed___ interval ___$[a, b]$___ and
2. f is differentiable on the ___open___ interval ___(a, b)___ .

Then there exists a number __c__ in __(a, b)__ such that

$$f'(c) = \underline{\qquad \frac{f(b)-f(a)}{b-a} \qquad}$$