Quiz 4 / Solutions

1. Compute the derivative of the following functions:

(a) $(x^3+1)^{\frac{1}{5}}$

Using the General Power Rule:

$$\begin{aligned}\left((x^3+1)^{\frac{1}{5}}\right)' &= \frac{1}{5}(x^3+1)^{(\frac{1}{5}-1)}\cdot(x^3+1)' \\ &= \frac{1}{5}(x^3+1)^{-\frac{4}{5}}\cdot 3x^2 \\ &= \boxed{\frac{3x^2}{5(x^3+1)^{\frac{4}{5}}}}\end{aligned}$$

(b) $\sin\left(\cos^3(x)\right)$

Using the Chain Rule and The General Power Rule:

$$\begin{aligned}\left(\sin\left(\cos^3(x)\right)\right)' &= \sin'\left(\cos^3(x)\right)\cdot\left(\cos^3(x)\right)' \\ &= \cos\left(\cos^3(x)\right)\cdot 3\cos^2(x)\cdot\cos'(x) \\ &= 3\cos\left(\cos^3(x)\right)\cdot\cos^2(x)\cdot(-\sin(x)) \\ &= \boxed{-3\cos\left(\cos^3(x)\right)\cdot\cos^2(x)\cdot\sin(x)}\end{aligned}$$

2. Let $s = t^4 + t^2 + 1$ be the position function of an object moving along a straight line.

(a) When does the object move forward?

We compute

$$v = s' = (t^4+t^2+1)' = 4t^3 + 2t = 2t(2t^2+1)$$

Note that $2t^2+1 > 0$ for all t. Hence $2t(2t^2+1) > 0$ if and only if $t > 0$. So the object is moving forward on the time interval

$$\boxed{(0,\infty)}$$

(b) What is the total distance traveled between $t = -1$ and $t = 1$.

From (a) we know that the object moves backwards from $t = -1$ to $t = 0$ and is moving forward from $t = 0$ to $t = 1$.

We compute

$$\begin{aligned}s(-1) &= (-1)^4 + (-1)^2 + 1 = 1+1+1 = 3 \\ s(0) &= 0^4 + 0^2 + 1 = 1 \\ s(1) &= 1^4 + 1^2 + 1 = 3\end{aligned}$$

The distance traveled on the interval $[-1,0]$ is $|s(0)-s(-1)| = |1-3| = |-2| = 2$.

The distance traveled on the interval $[0,1]$ is $|s(1)-s(0)| = |3-1| = |2| = 2$.
So the total distance traveled is $2+2 = \boxed{4}$.

3. Find the slope of the tangent line to the curve $xy^4 + y^3 = 2$ at the point $P(1,1)$.

Computing the derivative with respect to x on both sides of $xy^4 + y^3 = 2$ gives

$$\left(1 \cdot y^4 + x \cdot 4y^3 \cdot y'\right) + 3y^2 \cdot y' = 0$$

So

$$\begin{aligned} y^4 + 4xy^3y' + 3y^2y' &= 0 \\ y^4 + (4xy+3)y^2y' &= 0 \\ (4xy+3)y^2y' &= -y^4 \\ y' = \frac{-y^4}{(4xy+3)y^2} &= -\frac{y^2}{4xy+3} \end{aligned}$$

For $x = 1$ and $y = 1$ this gives

$$y' = -\frac{1^2}{4 \cdot 1 \cdot 1 + 3} = -\frac{1}{7}$$

Hence the slope of the tangent line to the curve $xy^4 + y^3 = 2$ at the point $P(1,1)$ is

$$\boxed{-\frac{1}{7}}.$$