MTH 132-020 **Calculus I** **F18**

Quiz 3 / Solutions

1. Compute the derivative of each of the following functions:

(a) $x^4 + \sqrt{x}$

Recall that $(x^n)' = nx^{n-1}$ So

$$(x^4 + \sqrt{x})' = (x^4)' + (x^{\frac{1}{2}})' = \boxed{4x^3 + \frac{1}{2}x^{-\frac{1}{2}}} = \boxed{4x^3 + \frac{1}{2\sqrt{x}}}.$$

□

(b) $x^2 \cos(x)$

$$(x^2 \cos(x)' = (x^2)' \cos(x) + x^2 \cos'(x) = \boxed{2x\cos(x) + x^2(-\sin x)} = \boxed{2x\cos(x) - x^2\sin(x)}$$

□

(c) $\frac{x^2}{\sin x}$

First solution:

$$(\frac{x^2}{\sin x})' = \frac{(x^2)'\sin(x) - x^2\sin'(x)}{(\sin x)^2} = \boxed{\frac{2x\sin(x) - x^2\cos(x)}{\sin^2(x)}}$$

Second solution:

$$(\frac{x^2}{\sin x})' = (x^2\csc(x))' = (x^2)'\csc(x) + x^2\csc'(x)$$
$$= \boxed{2x\csc(x) + x^2(-\csc(x)\cot(x))} = \boxed{2x\csc(x) - x^2\csc(x)\cot(x)}$$

□

2. Let d be a real number and let

$$f(x) = \begin{cases} \sin(x) & \text{if } x \le 0 \\ dx & \text{if } x > 0 \end{cases}$$

For which d is $f(x)$ differentiable at 0?

$f(x)$ is differentiable at 0 if and only if $\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ exists and so if and only if

$$(*) \qquad \lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0^-} \frac{f(x) - f(0)}{x - 0}$$

If $x \ge 0$, the $f(x) = dx$. So the left-hand side of (*) is the derivative of dx at $x = 0$ and so equal to d.

If $x \le 0$, the $f(x) = \sin(x)$. So the right-hand side of (*) is the derivative of $\sin(x)$ at 0 and so equal to $\cos(0) = 1$.

Hence $f(x)$ is differentiable at 0 if and only if $\boxed{d = 1}$. □

3. Compute $\lim_{x \to 0} \frac{\sin(3x)}{7}$.

Since $\sin(x)$ is continuous everywhere, also $\frac{\sin(3x)}{7}$ is continuous everywhere. So

$$\lim_{x \to 0} \frac{\sin(7x)}{3} = \frac{\sin(3 \cdot 0)}{7} = \frac{\sin(0)}{7} = \frac{0}{7} = \boxed{0}.$$

□

If you had to compute $\lim_{x \to 0} \frac{\sin(3x)}{7x}$*, the solution would be quite different:*

$$\lim_{x \to 0} \frac{\sin(3x)}{7x} = \lim_{x \to 0} \frac{7}{3} \frac{\sin(7x)}{7x} = \frac{7}{3} \lim_{x \to 0} \frac{\sin(7x)}{7x} = \frac{7}{3} \lim_{z \to 0} \frac{\sin(z)}{z} = \frac{7}{3} \cdot 1 = \frac{7}{3}.$$