Quiz 2 / Solutions

1. Fill in the blanks:

Definition: Let f be a __function__ defined on some __open__ interval that contains the number a, except possibly at __a__ itself. Then we say that the limit of $f(x)$ as x approaches a is L, and write

$$\lim_{x\to a} f(x) = \underline{\quad L \quad}$$

if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad 0 < |\underline{\ x-a\ }| < \delta \qquad \text{then} \qquad |\underline{\ f(x)\ } - L| < \underline{\quad \epsilon \quad}.$$

2. On which of the following intervals is the greatest integer function $[\![x]\!]$ continuous? Circle all correct answers.

(a) (0,2) $\boxed{(b)\quad [0,1)}$ $\boxed{(c)\quad (0,1)}$ (d) (0,1]

Recall from class that $[\![x]\!]$ is continuous at any non-integer, $[\![x]\!]$ is right continuous at all integers, and $[\![x]\!]$ is not left continuous at any integer.

Note that $0 < 1 < 2$. As $[\![x]\!]$ is not continuous at 1, this shows that $[\![x]\!]$ is $\boxed{\text{not continuous on } (0,2)}$.

Let a be a number with $0 < a < 1$. Then a is not an integer and so $[\![x]\!]$ is continuous at a. Also $[\![x]\!]$ is right continuous at the left endpoint 0 of $[0,1)$ and so $[\![x]\!]$ is $\boxed{\text{continuous on } [0,1)}$ and also $\boxed{\text{continuous on } (0,1)}$.

Since $[\![x]\!]$ is not left continuous are the right endpoint 1 of $(0,1]$, we see $[\![x]\!]$ is $\boxed{\text{not continuous on } (0,1]}$.

3. Show that there exists a number c in the open interval $(0, \frac{\pi}{2})$ such that $2c - \sin(c) = 0.2$.(I changed the problem from the actual quiz so that is has a solution)

Let $f(x) = 2x - \sin(x)$. Then $f(x)$ is continuous everywhere. Also

$$f(0) = 2\cdot 0 - \sin(0) = 0 - 0 = 0, \quad \text{and} \quad f(\frac{\pi}{2}) = 2\cdot\frac{\pi}{2} - \sin(\frac{\pi}{2}) = \pi - 1 \approx 3.14 - 1 = 2.14$$

Hence

$$f(0) < 0.2 < f(\frac{\pi}{2})$$

and so the Intermediate Value Theorem shows that there exists c in the interval $(0, \frac{\pi}{2})$ such that $f(c) = 0.2$. Then

$$c - \sin(c) = 0.2.$$

4. Use the definition of the derivative to compute the derivative of $x^2 - 7$ at 2.

Let $f(x) = x^2 - 7$. Then

$$
\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h\to 0} \frac{((2+h)^2 - 7) - (2^2 - 7)}{h} \\
&= \lim_{h\to 0} \frac{((2^2 + 2\cdot 2\cdot h + h^2) - 7) - (2^2 - 7)}{h} \\
&= \lim_{h\to 0} \frac{(4 + 4h + h^2 - 7) - (4 - 7)}{h} \\
&= \lim_{h\to 0} \frac{4h + h^2}{h} \\
&= \lim_{h\to 0} \frac{(4+h)h}{h} \\
&= \lim_{h\to 0} 4 + h \\
&= 4 + 0 \\
&\boxed{= 4}
\end{aligned}
$$