Quiz 1 / Solutions

Each answer needs to be a real number, $+\infty$, $-\infty$ or DNE)

1. $\lim_{x\to 0^-} -\frac{1}{x^2}$

If x is a small negative number, then x^2 is a small positive number. So $\frac{1}{x^2}$ is a large positive number and $-\frac{1}{x^2}$ is a large negative number. Thus

$$\lim_{x\to 0^-} -\frac{1}{x^2} = \boxed{-\infty}.$$

2. Let

$$f(x) = \begin{cases} x^3 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad x = 0 \\ \sin\left(\frac{1}{x}\right) & \text{if} \quad x > 0 \end{cases}$$

(a) $\lim_{x\to 0^+} f(x)$

If $x > 0$, then $f(x) = \sin\left(\frac{1}{x}\right)$. So

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sin\left(\frac{1}{x}\right).$$

As x approaches 0 from the right, $\frac{1}{x}$ approaches $+\infty$. Note that $\sin t$ oscillates infinitely often between -1 and 1 as t approaches $+\infty$. So, as x approaches 0 from the right, $\sin\left(\frac{1}{x}\right)$ oscillates infinitely often between -1 and 1. Thus $\lim_{x\to 0+} \sin\left(\frac{1}{x}\right)$ does not exist. So

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sin\left(\frac{1}{x}\right) = \boxed{\text{DNE}}$$

(b) $\lim_{x\to 0^-} f(x)$

If $x < 0$, then $f(x) = x^3$. So

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^3 = 0^3 = \boxed{0}$$

3. $\lim_{x\to 2} \frac{x-2}{x^2-x-2}$

$$\lim_{x\to 2} \frac{x-2}{x^2-x-2} = \lim_{x\to 2} \frac{x-2}{(x-2)(x+1)} = \lim_{x\to 2} \frac{1}{x+1} = \lim_{x\to 2} \frac{1}{2+1} = \boxed{\frac{1}{3}}.$$

4. $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{(\sqrt{x})^3 - 3^2}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}.$$