

**Standard Response Questions.** Show all your work to receive credit. Please **BOX** your final answer.

#1. (6 pts) Find the most general antiderivative of  $f(x) = x^5 - \sec(x) \tan(x) + \frac{1}{2\sqrt{x}}$ .

**Solution:**

$$\frac{1}{6}x^6 - \sec x + \sqrt{x} + C$$

#2. (8 pts) Determine the value(s) of  $a$  such that

$$\int_a^{a+1} (2x + 3) dx = 10$$

**Solution:**

The definite integral is the area of the region between  $y = 0$  and  $y = 2x + 3$  from  $x = a$  to  $x = a + 1$ . This area is a trapezoid of height 1 and bases  $2a + 3$  and  $2(a + 1) + 3$  and so has area

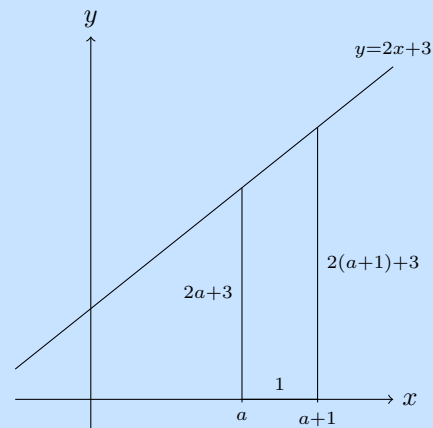
$$\frac{(2a + 1) + (2(a + 1) + 3)}{2} \cdot 1 = \frac{4a + 8}{2} = 2a + 4$$

So  $\int_a^{a+1} (2x + 3) dx = 2a + 4$  and we need to find a number  $a$  such that

$$2a + 4 = 10$$

$$2a = 6$$

$$\boxed{a = 3}$$



**Remark:** We could also have calculated the definite integral using the Fundamental Theorem of Calculus:

$$\begin{aligned} \int_a^{a+1} (2x + 3) dx &= [x^2 + 3x]_a^{a+1} \\ &= ((a + 1)^2 + 3(a + 1)) - (a^2 + 3a) \\ &= \cancel{a^2} + 2a + 1 + \cancel{3a} + 3 - \cancel{a^2} - \cancel{3a} \\ &= 2a + 4 \end{aligned}$$

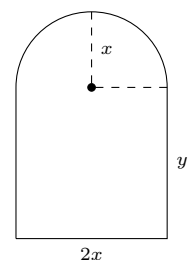
#3. (14 pts)

A small region has the shape of a rectangle attached to a semicircle, so that the diameter of the semicircle is equal to the width of the rectangle. The perimeter is 2 m.

What is the width of such a region which has the largest possible area?

What is the largest possible area?

Use one of the techniques of MTH 132 to justify that your solution indeed maximizes the area.



**Solution:** The area of the rectangle is  $2xy$  and the area of the semicircle is  $\frac{1}{2}\pi x^2$ . So the total area is

$$A = 2xy + \frac{\pi}{2}x^2.$$

The perimeter consists of the bottom (of length  $2x$ ), the two sides (each of length  $y$ ) and the semicircle (of perimeter  $\frac{1}{2} \cdot 2\pi x$ ). Hence the total perimeter is

$$P = 2x + 2y + \pi x$$

We know that  $P = 2$ , so

$$2 = 2x + 2y + \pi x$$

$$2y = 2 - 2x + \pi x$$

$$y = 1 - x + \frac{\pi}{2}x$$

Substituting in the formula for  $A$  gives

$$\begin{aligned} A &= 2x \left( 1 - x + \frac{\pi}{2}x \right) + \frac{\pi}{2}x^2 \\ &= 2x - 2x^2 - \pi x^2 + \frac{\pi}{2}x^2 \\ &= 2x - 2x^2 - \frac{\pi}{2}x^2 \end{aligned}$$

Thus

$$\begin{aligned} A' &= 2 - 4x - \pi x \\ &= 2 - (4 + \pi)x \\ &= (4 + \pi) \left( \frac{2}{4 + \pi} - x \right) \end{aligned}$$

It follows that  $A' = 0$  for  $x = \frac{2}{4 + \pi}$ . Moreover

$$A' > 0 \text{ for } x < \frac{2}{4 + \pi} \quad \text{and} \quad A' < 0 \text{ for } x > \frac{2}{4 + \pi}.$$

Thus the First Derivative Test for Absolute Extrema shows that  $A$  has an absolute maximum at  $x = \frac{2}{4 + \pi}$ . The width of the region with maximal area is

$$2x = 2 \frac{2}{4 + \pi} = \boxed{\frac{4}{4 + \pi} \text{ m}}.$$

and the maximal area is

$$\begin{aligned}
A &= 2x - 2x^2 - \frac{\pi}{2}x^2 \\
&= \boxed{2\frac{2}{4+\pi} - 2\left(\frac{2}{4+\pi}\right)^2 - \frac{\pi}{2}\left(\frac{2}{4+\pi}\right)^2} \\
&= \frac{4}{4+\pi} - \frac{8}{(4+\pi)^2} - \frac{2\pi}{(4+\pi)^2} \\
&= \frac{(16+4\pi) - 8 - 2\pi}{(4+\pi)^2} \\
&= \frac{8+2\pi}{(4+\pi)^2} \\
&= \frac{\cancel{2(4+\pi)}}{(4+\pi)^{\cancel{2}}} \\
&= \boxed{\frac{2}{4+\pi} \text{ m}^2}
\end{aligned}$$

#4. (7 pts) Given  $f(x) = 5x^{2/3} - 2x^{5/3}$

(a) (4 pts) Determine all its critical points.

**Solution:**

$$\begin{aligned}
f'(x) &= \left(5x^{2/3} - 2x^{5/3}\right)' \\
&= \frac{2}{3} \cdot 5x^{-1/3} - \frac{5}{3} \cdot 2x^{2/3} \\
&= \frac{10}{3} \left(x^{-1/3} - x^{2/3}\right) \\
&= \frac{10}{3} \frac{1}{x^{1/3}} - x^{2/3} \\
&= \frac{10}{3} \frac{1 - x^{1/3}x^{2/3}}{x^{1/3}} \\
&= \frac{10}{3} \frac{1 - x}{x^{1/3}}
\end{aligned}$$

Thus  $f' = 0$  at  $x = 1$  and  $f'(x)$  is not defined at  $x = 0$ . Note that both  $x = 0$  and  $x = 1$  are in the domain of  $f$ , thus the critical numbers are

$$\boxed{x = 0} \quad \text{and} \quad \boxed{x = 1}$$

(b) (4 pts) Classify the critical points as local minima/maxima/neither.

**Solution:**

	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
$1 - x$	+	+	-
$x^{1/3}$	-	+	+
$f'$	-	+	-
$f$	$\searrow$	$\nearrow$	$\searrow$

Thus  $f$  has a local minimum at  $x = 0$  and a local maximum at  $x = 1$

#5. (7 pts) Determine the absolute extrema of the function  $f(x) = x - 2 \sin x$  on the interval  $[0, \pi]$ . ( $\sqrt{3} \approx 1.73$ )

**Solution:** Note that  $f(x)$  is continuous, so we can use the Continuous Closed Interval Method.

$$f'(x) = (x - 2 \sin x)' = 1 - 2 \cos x$$

Hence  $f'$  is defined for all  $x$  and

$$\begin{aligned} f'(x) &= 0 \\ 1 - 2 \cos x &= 0 \\ 2 \cos x &= 1 \\ \cos x &= \frac{1}{2} \end{aligned}$$

In the interval  $[0, \pi]$  the only solution of  $\cos x = \frac{1}{2}$  is  $x = \frac{\pi}{3}$ . Thus  $\frac{\pi}{3}$  is the only critical number. Next we compute the values of  $f$  at the endpoints and the critical points:

$$\begin{aligned} f(0) &= 0 - 2 \sin(0) = 0 - 2 \cdot 0 &= 0 \\ f\left(\frac{\pi}{3}\right) &= \frac{\pi}{3} - 2 \sin \frac{\pi}{3} = \frac{\pi}{3} - 2 \frac{\sqrt{3}}{2} < \frac{3.3}{3} - 1.7 = 1.1 - 1.7 = -0.6 < 0 \\ f(\pi) &= \pi - 2 \sin \pi = \pi - 2 \cdot 0 = \pi &> 0 \end{aligned}$$

The largest of these values is  $\pi$ , and the smallest is  $\frac{\pi}{3} - \sqrt{3}$ . Thus the absolute maximum value of  $f$  on  $[0, \pi]$  is  $\pi$ , attained at  $x = \pi$ , and the absolute minimum value of  $f$  on  $[0, \pi]$  is  $\frac{\pi}{3} - \sqrt{3}$ , attained at  $x = \frac{\pi}{3}$ .

#6. (6 pts) Compute  $\lim_{x \rightarrow \infty} \left( \sqrt{4x^2 + 3x} - 2x \right)$ .

**Solution:** We first compute

$$\begin{aligned} \sqrt{4x^2 + 3x} - 2x &= \frac{\sqrt{4x^2 + 3x} - 2x}{1} \cdot \frac{\sqrt{4x^2 + 3x} + 2x}{\sqrt{4x^2 + 3x} + 2x} = \frac{\sqrt{4x^2 + 3x}^2 - (2x)^2}{\sqrt{4x^2 + 3x} + 2x} \\ &= \frac{4x^2 + 3x - 4x^2}{\sqrt{4x^2 + 3x} + 2x} = \frac{3x}{\sqrt{x^2 \left(4 + \frac{3}{x}\right)} + 2x} \\ &= \frac{3x}{x \sqrt{4 + \frac{3}{x}} + 2x} = \frac{3\cancel{x}}{\cancel{x} \left( \sqrt{4 + \frac{3}{x}} + 2 \right)} \\ &= \frac{3}{\sqrt{4 + \frac{3}{x}} + 2} \end{aligned}$$

and so

$$\lim_{x \rightarrow \infty} \sqrt{4x^2 + 3x} - 2x = \lim_{x \rightarrow \infty} \frac{3}{\sqrt{4 + \frac{3}{x} + 2}} = \frac{3}{\sqrt{4 + 0 + 2}} = \frac{3}{2 + 2} = \boxed{\frac{3}{4}}$$

#7. (8 pts) The acceleration of an object moving along the  $x$ -axis is  $a(t) = 3 \sin t$ . What are its velocity and position functions,  $v(t)$  and  $s(t)$ , if  $v(0) = 1$  and  $s(0) = 3$ ?

**Solution:** Recall that  $a = v'$ , so  $v$  is an antiderivative of  $3 \sin t$ . Thus

$$v = -3 \cos t + C,$$

$C$  a constant. This gives

$$1 = v(0) = -3 \cos 0 + C = -3 \cdot 1 + C = C - 3$$

and so  $C = 1 + 3 = 4$ . Hence

$$\boxed{v = -\cos t + 4}$$

and since  $s' = v$ ,

$$s = -3 \sin t + 4t + D,$$

$D$  a constant. So

$$3 = s(0) = -3 \sin 0 + 4 \cdot 0 = -0 + 0 + D = D$$

and

$$s = \boxed{-3 \sin t + 4t + 3}.$$

**Multiple Choice.** Circle the best answer. No work needed.  
No partial credit available.

#8. (4 pts) Determine all values of  $c$  satisfying the Mean Value Theorem for the function  $f(x) = x^3 - 4x$  on the interval  $-1 \leq x \leq 3$ .

A.  $\left(\frac{7}{3}\right)^{1/2}$

B.  $\pm\sqrt{\frac{7}{3}}$

C. 5

D. 3

**Solution 1:** Recall that the number  $c$  in the Mean Value Theorem lies in the open interval  $(-1, 3)$ . Note that  $1 < \frac{7}{3} < 4$  and so  $1 < \sqrt{\frac{7}{3}} < 2$  and  $-\sqrt{\frac{7}{3}} < -1$ . So of the given possible answers only  $\sqrt{\frac{7}{3}}$  (which is equal to  $(\frac{7}{3})^{1/2}$ ) lies in  $(-1, 3)$ . Hence A. is the only possible correct answer.

**Solution 2:** According to the Mean Value Theorem,  $c$  is in  $(a, b)$  and

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

We have  $a = -1$ ,  $b = 3$ ,

$$\begin{aligned}f(x) &= x^3 - 4x \\f'(x) &= 3x^2 - 4 \\f(3) &= 3^3 - 4 \cdot 3 = 27 - 12 = 15 \\f(-1) &= (-1)^3 - 4 \cdot (-1) = -1 + 4 = 3\end{aligned}$$

$$\begin{aligned}3c^2 - 4 = f'(c) &= \frac{f(3) - f(-1)}{3 - (-1)} = \frac{15 - 3}{4} = \frac{12}{4} = 3 \\3c^2 &= 3 + 4 = 7 \\c^2 &= \frac{7}{3} \\c &= \pm \sqrt{\frac{7}{3}}\end{aligned}$$

Of  $\sqrt{\frac{7}{3}}$  and  $-\sqrt{\frac{7}{3}}$  only  $\sqrt{\frac{7}{3}}$  lies in the interval  $(-1, 3)$ . So  $c = \sqrt{\frac{7}{3}} = (\frac{7}{3})^{1/2}$  and A. is the correct answer.

#9. (4 pts) If the length of a side of a cube is measured to be 5 cm with a maximum error of 0.1 cm, use differentials to estimate the maximum error in the surface area.

- A.  6 cm<sup>2</sup>      B.  11.2 cm<sup>2</sup>      C.  3 cm<sup>2</sup>      D.  60 cm<sup>2</sup>      E.  6 cm

**Solution:** Let  $x$  be the length of the side of the cube and  $S$  the surface area. The surface area consists of six squares of height  $x$ . So  $S = 6x^2$ . Thus

$$dS = \frac{dS}{dx} dx = \frac{d6x^2}{dx} dx = 12x dx$$

For  $x = 5$  and  $dx = \Delta x = 0.1$  we get

$$dS = 12 \cdot 5 \cdot 0.1 = 60 \cdot 0.1 = 6 \text{ cm}^2$$

Thus A. is the correct answer.

#10. (4 pts) Compute  $\lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} \frac{2i}{n^2}$ .

- A.  0      B.  1      C.  2      D.  DNE

**Solution 1:**

$$\begin{aligned}\sum_{i=1}^{\infty} \frac{2i}{n^2} &= \frac{2}{n^2} \sum_{i=1}^n i \\&= \frac{\cancel{2} \cdot \cancel{n}(n-1)}{n^{\cancel{2}} \cdot \cancel{2}} \\&= \frac{n-1}{n} \\&= 1 - \frac{1}{n}\end{aligned}$$

and so

$$\lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} \frac{2i}{n^2} = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right) = 1 - 0 = 1$$

So B. is the correct answer.

**Solution 2:** We will write the limit as a limit of right hand sums  $R_n = \sum_{i=1}^n f(x_i)\Delta x$ . Recall that  $x_i = a + i\Delta x$  and  $\Delta x = \frac{b-a}{n}$ . So we want to find  $f$ ,  $\Delta x$ ,  $a$  and  $b$  such that

$$\begin{aligned} & f(a + i\Delta x)\Delta x \\ &= \frac{2i}{n^2} \\ &= 2i \frac{1}{n} \frac{1}{n} \end{aligned}$$

So we can choose  $f(x) = 2x$ ,  $a = 0$  and  $\Delta x = \frac{1}{n}$ . Then

$$b = a + n\Delta x = 0 + n \frac{1}{n} = 1$$

Thus

$$\begin{aligned} \lim_{n \rightarrow \infty} \sum_{i=1}^{\infty} \frac{2i}{n^2} &= \lim_{x \rightarrow \infty} \sum_{i=1}^n f(x_i)\Delta x \\ &= \int_a^b f(x) dx \\ &= \int_0^1 2x dx \\ &= [x^2]_0^1 \\ &= 1^2 - 0^2 \\ &= 1 \end{aligned}$$

Hence B. is the correct answer.

#11. (4 pts) Which function should you apply Newton's method to, in order to estimate  $\sqrt{5}$ ?

A.  $x^2 - 25$

B.  $\sqrt{5} - x^2$

C.  $x - 5$

D.  $x^2 - 5$

**Solution:** We need a function  $f$  with  $f(\sqrt{5}) = 0$ . This holds for  $f = x^2 - 5$ . So D. is a correct answer. To make sure that none of the others work:

$$\begin{aligned} (\sqrt{5})^2 - 25 &= 5 - 25 \neq 0 \\ \sqrt{5} - (\sqrt{5})^2 &= \sqrt{5} - 5 \neq 0 \\ \sqrt{5} - 5 &\neq 0 \end{aligned}$$

So D. is indeed the only correct answer.

#12. (4 pts) The derivative of  $f(x) = \int_1^{2x^2} \frac{\sin t}{1+t^2} dt$  is

A.  $\frac{\sin x}{1+x^2}$

B.  $\frac{\sin(2x^2)}{1+4x^4}$

C.  $\frac{4x \sin x}{1+x^2}$

D.  $\frac{4x \sin(x^2)}{1+4x^4}$

**Solution:** Let

$$g(x) = \frac{\sin x}{1+x^2} \quad \text{and} \quad G(x) = \int_1^x g(t) dt = \int_1^x \frac{\sin t}{1+t^2} dt.$$

By the Fundamental Theorem of Calculus  $G'(x) = g(x)$ . So

$$\begin{aligned} \left( \int_1^{2x^2} \frac{\sin t}{1+t^2} dt \right)' &= (G(2x^2))' \\ &= G'(2x^2)(2x^2)' \\ &= g(2x^2)4x \\ &= \frac{\sin(2x^2)}{1+(2x^2)^2}4x \\ &= \frac{4x \sin(2x^2)}{1+4x^4} \end{aligned}$$

Hence D. is the correct answer.

#13. (4 pts) Determine the value of  $\int_{-5}^0 |x+3| dx$ . (Hint: Draw a picture of the region the integral represents, and find the area using simple formulas from geometry.)

A. -6.5

B. -5.5

C. 0.5

D. 5.5

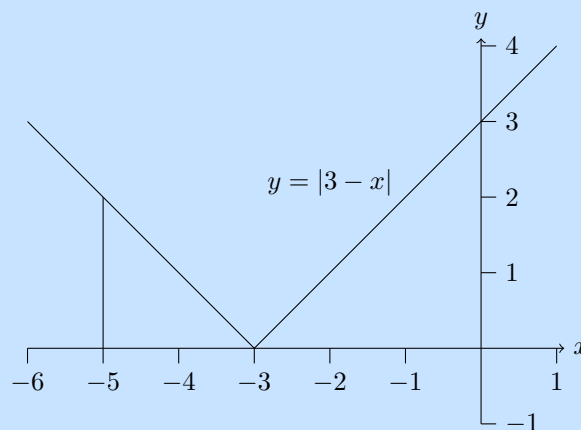
E. 6.5

**Solution:**

The definite integral is the area of the region between  $y = 0$  and  $y = |x+3|$  from  $x = -5$  to  $x = 0$ . This region consists of two triangles: One of height 2 and base 2 and one of height 3 and base 3. The area of a triangle with height  $h$  and base  $b$  is  $\frac{1}{2}hb$ . So

$$\int_{-5}^0 |x+3| dx = \frac{1}{2} \cdot 2 \cdot 2 + \frac{1}{2} \cdot 3 \cdot 3 = \frac{4+9}{2} = \frac{13}{2} = 6.5$$

Hence E. is the correct answer.



#14. (4 pts) Using linear approximation, what is the best estimate of  $\sqrt{4.1}$ ?

A.  $2 + \frac{1}{40}$

B.  $2 + \frac{1}{20}$

C.  $2 + \frac{1}{10}$

D. 2.



**Solution:** Let  $f(x) = \sqrt{x}$  and  $a = 4$ . Then

$$\begin{aligned} f(4) &= \sqrt{4} = 2 \\ f'(x) &= \frac{1}{2\sqrt{x}} \\ f'(4) &= \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4} \\ L(x) &= f(a) + f'(a)(x - a) \\ &= 2 + \frac{1}{4}(x - 4) \end{aligned}$$

and so

$$\sqrt{4.1} \approx L(4.1) = 2 + \frac{1}{4}(4.1 - 4) = 2 + \frac{1}{4} \cdot 0.1 = 2 + \frac{1}{4} \cdot \frac{1}{10} = 2 + \frac{1}{40}$$

Hence A. is the correct answer.

#15. (4 pts) Select the true statements about the function  $f(x) = \frac{x^3+4x}{(x+2)(x-1)}$ :

- A. The function has no vertical asymptotes and only one slant asymptote.
- B. The function has only one vertical asymptote and only one slant asymptote.
- C. The function has only two vertical asymptote and no slant asymptotes.
- D. The function has only two vertical asymptote and only one slant asymptote.

**Solution:** Since  $f(x) = \frac{x^3+x}{(x+2)(x-1)} = \frac{x(x^2+1)}{(x+2)(x-1)}$  we see that  $x = -2$  and  $x = 1$  are the zeros of the denominator of  $f$ , but neither  $-2$  nor  $1$  are zeros of the numerator of  $f$ . It follows that

$$\lim_{x \rightarrow 2^\pm} f(x) = \pm\infty \quad \text{and} \quad \lim_{x \rightarrow -1^\pm} f(x) = \pm\infty$$

Thus

$$x = 2 \quad \text{and} \quad x = -1$$

are the vertical asymptotes of  $f(x)$ .

Note that  $(x-2)(x+1) = x^2 - 2x + x - 2 = x^2 - x - 2$  and

$$\begin{array}{r} x^2 - x - 2 \overline{) \begin{array}{r} x^3 \quad + \quad 1 \\ x^3 \quad - \quad x^2 \quad - \quad 2x \\ \hline x^2 \quad + \quad 3x \\ x^2 \quad - \quad x \quad - \quad 2 \\ \hline 4x \quad + \quad 2 \end{array}} \end{array}$$

So

$$f(x) = x + 1 + \frac{4x + 2}{x^2 - x - 2} = x + 1 + \frac{x(4 + \frac{2}{x})}{x^2(1 - \frac{1}{x} - \frac{2}{x^2})} = \frac{1 + \frac{2}{x}}{x(1 - \frac{1}{x} - \frac{2}{x^2})}$$

and

$$\lim_{x \rightarrow \pm\infty} f(x) - (x + 1) = \lim_{x \rightarrow \pm\infty} \frac{1 + \frac{2}{x}}{x(1 - \frac{1}{x} - \frac{2}{x^2})} = 0$$

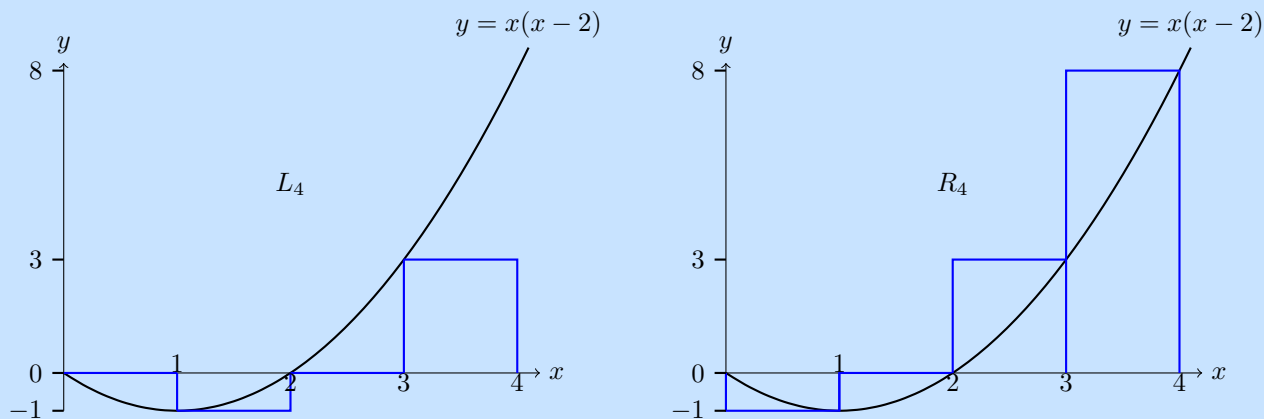
Thus  $x + 1$  is the only slant asymptote of  $f(x)$ . So D. is the correct answer.

#16. (4 pts) Estimate the area  $A$  under the graph  $y = x(x - 2)$ , between  $x = 0$  and  $x = 4$ , using 4 rectangles of equal width, with heights of the rectangles determined by the height of the curve at the left endpoints and the right endpoints.

- A.  $A = 1$  using left endpoints;  $A = 8$  using right endpoints.  
 B.  $A = -1$  using left endpoints;  $A = 9$  using right endpoints.  
 C.  $A = 10$  using left endpoints;  $A = 2$  using right endpoints.

D.  $A = 2$  using left endpoints;  $A = 10$  using right endpoints.

**Solution:**



Each interval has length  $\frac{4-0}{4} = 1$ . So the partition points are 0, 1, 2, 3, 4. We compute

$x$	0	1	2	3	4
$x(x - 2)$	0	-1	0	3	8

Thus

$$L_4 = (0 + (-1) + 0 + 3) \cdot 1 = 2$$

$$R_4 = (-1 + 0 + 3 + 8) \cdot 1 = 10$$

Thus D. is the correct answer.

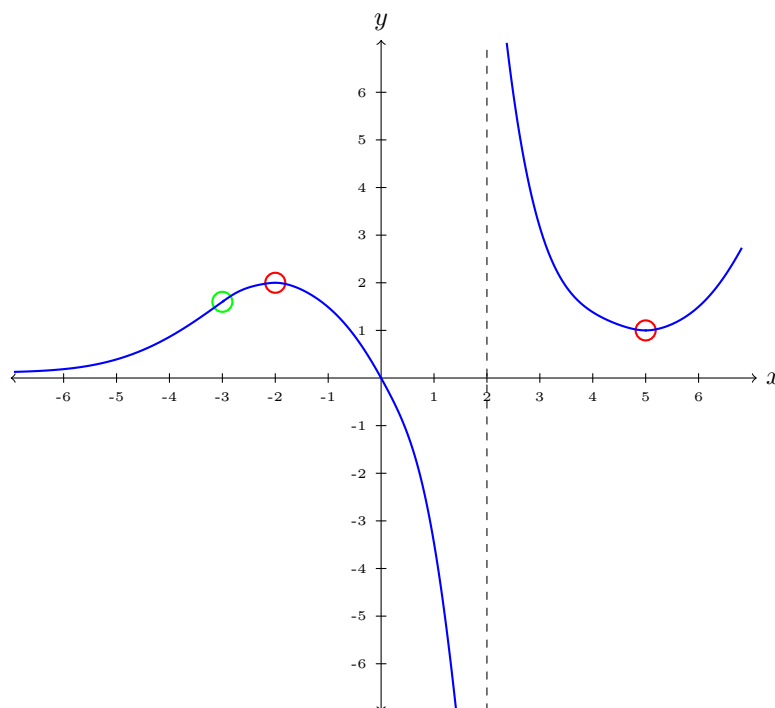
More Challenging Question(s)

#17. (14 pts) The function  $f(x)$  has *all* of the following properties.

- $\lim_{x \rightarrow 2^-} f(x) = -\infty$
- $\lim_{x \rightarrow 2^+} f(x) = \infty$
- $f(2)$  DNE.
- $\lim_{x \rightarrow -\infty} f(x) = 0$ .
- $f(-2) = 2$
- $f(5) = 1$
- $f(0) = 0$ .
- $f'(x) > 0$  if  $x < -2$  or  $x > 5$ .
- $f'(x) < 0$  if  $-2 < x < 2$  or  $2 < x < 5$ .
- $f'(5) = 0$ .
- $f'(-2) = 0$
- $f''(x) > 0$  if  $x < -3$  or  $x > 2$ .
- $f''(x) < 0$  if  $-3 < x < 2$ .

Complete the following sentences:

- (a) The domain of the function is:  $(-\infty, -2) \cup (2, \infty)$
- (b) Such function must have vertical asymptote(s), with equation(s):  $x = 2$ .
- (c) There must be a horizontal asymptote with equation  $y = 0$ .
- (d) There must be a local maximum of  $2$  at  $x = -2$ , and a local minimum of  $1$  at  $x = 5$ .
- (e) Such function must have inflection point(s) at  $x = -3$ .
- (f) The function must be negative on  $(0, 2)$ .
- (g) Sketch the curve.



**Solution:** Since  $f(2)$  does not exist, 2 is not in the domain. We know that  $f' > 0$  for  $x < -2$ ,  $f'(-2) = 0$ ,  $f' > 0$  for  $-2 < x < 2$ ,  $f'(x) < 0$  for  $2 < x < 5$ ,  $f'(5) = 0$  and  $f'(x) > 0$  for  $5 < x$ . Thus  $f'(x)$  is defined for all  $x \neq 2$  and so also  $f$  is defined for all  $x \neq 2$ . Thus the domain of  $f$  is  $(-\infty, 2) \cup (2, \infty)$ .

Since  $\lim_{x \rightarrow 2^{\pm 2}} f(x) = \pm\infty$ , we know that  $x = 2$  is a vertical asymptote.

As  $\lim_{x \rightarrow -\infty} f(x) = 0$ , we know that  $y = 0$  is a horizontal asymptote.

We have

	$(-\infty, -2)$	$(-2, 2)$	$(2, 5)$	$(5, \infty)$
$f'$	+	-	-	+
$f$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

Also  $f(-2) = 2$  and  $f(5) = 1$ . So the first derivative test shows that  $f$  has a local maximum value of 2 at  $x = -2$  and a local minimum value of 1 at  $x = 5$ .

From the information on the second derivative we have

	$(-\infty, -3)$	$(-3, 2)$	$(5, \infty)$
$f''$	+	-	+
$f$	∪	∩	∪

Hence  $f$  changes concavity at  $x = -3$  and  $x = 2$ . But  $f$  is not defined at  $x = 2$ . So the only inflection point of  $f$  is at  $x = -3$ .

Since  $f(0) = 0$  and  $f$  is decreasing on  $(-2, 2)$  we see that  $f$  is negative on  $(0, 2)$ .