

**Standard Response Questions.** Show all your work to receive credit. Please **BOX** your final answer.

#1. (6 pts) Find the most general antiderivative of  $f(x) = x^5 - \sec(c) \tan(x) + \frac{1}{2\sqrt{x}}$ .

#2. (8 pts) Determine the value(s) of  $a$  such that

$$\int_a^{a+1} (2x + 3) dx = 10$$

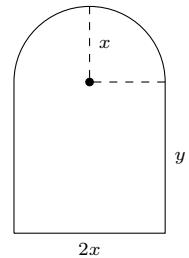
#3. (14 pts)

A small region has the shape of a rectangle attached to a semicircle, so that the diameter of the semicircle is equal to the width of the rectangle. The perimeter is 2 m.

What is the width of such a region which has the largest possible area?

What is the largest possible area?

Use one of the techniques of MTH 132 to justify that your solution indeed maximizes the area.



#4. (7 pts) Given  $f(x) = 5x^{2/3} - 2x^{5/3}$

(a) (4 pts) Determine all its critical points.

(b) (4 pts) Classify the critical points as local minima/maxima/neither.

#5. (7 pts) Determine the absolute extrema of the function  $f(x) = x - 2 \sin x$  on the interval  $[0, \pi]$ . ( $\sqrt{3} \approx 1.73$ )

#6. (6 pts) Compute  $\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x} - 2x)$ .

#7. (8 pts) The acceleration of an object moving along the  $x$ -axis is  $a(t) = 3 \sin t$ . What are its velocity and position functions,  $v(t)$  and  $s(t)$ , if  $v(0) = 1$  and  $s(0) = 3$ ?

**Multiple Choice.** Circle the best answer. No work needed.  
No partial credit available.

#8. (4 pts) Determine all values of  $c$  satisfying the Mean Value Theorem for the function  $f(x) = x^3 - 4x$  on the interval  $-1 \leq x \leq 3$ .

- A.  $(\frac{7}{3})^{1/2}$                       B.  $\pm\sqrt{\frac{7}{3}}$                       C. 5                      D. 3

#9. (4 pts) If the length of a side of a cube is measured to be 5 cm with a maximum error of 0.1 cm, use differentials to estimate the maximum error in the surface area.

- A.  $6 \text{ cm}^2$                       B.  $11.2 \text{ cm}^2$                       C.  $3 \text{ cm}^2$                       D.  $60 \text{ cm}^2$                       E.  $6 \text{ cm}$

#10. (4 pts) Compute  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2i}{n^2}$ .

- A. 0                      B. 1                      C. 2                      D. DNE

#11. (4 pts) Which function should you apply Newton's method to, in order to estimate  $\sqrt{5}$ ?

- A.  $x^2 - 25$                       B.  $\sqrt{5} - x^2$                       C.  $x - 5$                       D.  $x^2 - 5$

#12. (4 pts) The derivative of  $f(x) = \int_1^{2x^2} \frac{\sin t}{1+t^2} dt$  is

- A.  $\frac{\sin x}{1+x^2}$                       B.  $\frac{\sin(2x^2)}{1+4x^4}$                       C.  $\frac{4x \sin x}{1+x^2}$                       D.  $\frac{4x \sin(x^2)}{1+4x^4}$

#13. (4 pts) Determine the value of  $\int_{-5}^0 |x+3| dx$ . (*Hint: Draw a picture of the region the integral represents, and find the area using simple formulas from geometry.*)

- A. -6.5                      B. -5.5                      C. 0.5                      D. 5.5                      E. 6.5

#14. (4 pts) Using linear approximation, what is the best estimate of  $\sqrt{4.1}$ ?

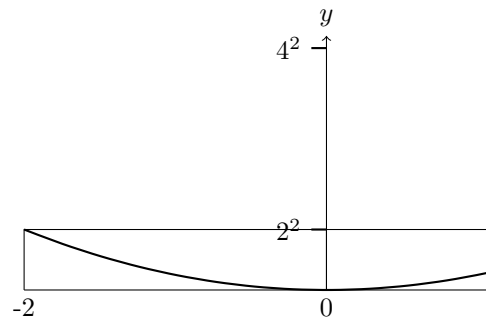
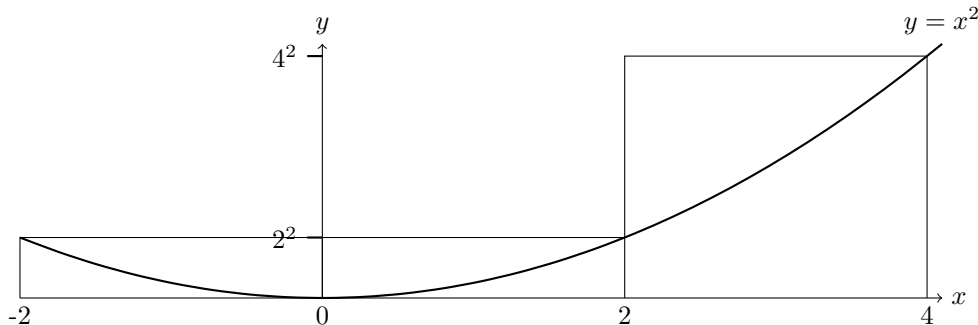
- A.  $2 + \frac{1}{40}$                       B.  $2 + \frac{1}{20}$                       C.  $2 + \frac{1}{10}$                       D. 2.

#15. (4 pts) Select the true statements about the function  $f(x) = \frac{x^3+4x}{(x+2)(x-1)}$ :

- A. The function has no vertical asymptotes and only one slant asymptote.
- B. The function has only one vertical asymptote and only one slant asymptote.
- C. The function has only two vertical asymptote and no slant asymptotes.
- D. The function has only two vertical asymptote and only one slant asymptote.

#16. (4 pts) Estimate the area  $A$  under the graph  $y = x(x-2)$ , between  $x = 0$  and  $x = 4$ , using 4 rectangles of equal width, with heights of the rectangles determined by the height of the curve at the left endpoints and the right endpoints.

- A.  $A = 1$  using left endpoints;  $A = 8$  using right endpoints.
- B.  $A = -1$  using left endpoints;  $A = 9$  using right endpoints.
- C.  $A = 10$  using left endpoints;  $A = 2$  using right endpoints.
- D.  $A = 2$  using left endpoints;  $A = 10$  using right endpoints.



More Challenging Question(s)

#17. (14 pts) The function  $f(x)$  has *all* of the following properties.

- |  |   |
|--|---|
| 1. $\lim_{x \rightarrow 2^-} f(x) = -\infty$ | 7. $f(0) = 0$ .                                 |
| 2. $\lim_{x \rightarrow 2^+} f(x) = \infty$  | 8. $f'(x) > 0$ if $x < -2$ or $x > 5$ .         |
| 3. $f(2)$ DNE.                               | 9. $f'(x) < 0$ if $-2 < x < 2$ or $2 < x < 5$ . |
| 4. $\lim_{x \rightarrow -\infty} f(x) = 0$ . | 10. $f'(5) = 0$ .                               |
| 5. $f(-2) = 1$                               | 11. $f'(-2) = 0$                                |
| 6. $f(5) = 1$                                | 12. $f''(x) > 0$ if $x < -3$ or $x > 2$ .       |
|  | 13. $f''(x) < 0$ if $-3 < x < 2$ .              |

Complete the following sentences:

- (a) The domain of the function is: \_\_\_\_\_
- (b) Such function must have vertical asymptote(s), with equation(s): \_\_\_\_\_
- (c) There must be a horizontal asymptote with equation \_\_\_\_\_
- (d) There must be a local maximum of \_\_\_\_\_, and a local minimum of \_\_\_\_\_
- (e) Such function must have inflection point(s) at  $x =$  \_\_\_\_\_.
- (f) The function must be negative on \_\_\_\_\_.
- (g) Sketch the curve.

