MTH 132 **Exam 1** **Fall 2018**

Standard Response Questions. Error Carried Forward

#1. (9 pts) Calculate the following limits or show that they do no exist:

(a) (4 pts) $\lim_{x\to -1} \dfrac{x^2-1}{x+1} =$

(b) (5 pts) $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right) =$

#2. (5 pts) Find the value of a that makes the function continuous at $x = 0$:

$$f(x) = \begin{cases} \frac{\sin(-8x)}{x} & \text{if } x < 0 \\ 3x + 6a - 7 & \text{if } x \geq 0 \end{cases}$$

#3. (7 pts) The length of a rectangle is decreasing at a rate of 4cm/s and its width is increasing at a rate of 5cm/s. When the length is 12cm and the width 10cm, how fast is the area of the rectangle changing? Is the area increasing or decreasing at that time? (*Include units*)

#4. (7 pts) Given $f(x) = x^2 + 10\sin x$.

(a) (1 pts) Indicate the interval where the function is continuous.

(b) (4 pts) Prove that there is a number c such that $f(c) = 1$, using the Intermediate Value Theorem.

(c) (2 pts) Using (b) state an interval where c can be found.

#5. (8 pts) Compute the derivative of the following functions: (**DO NOT SIMPLIFY**)

(a) (4 pts) $f(x) = x\sec x$

(b) (4 pts) $g(x) = \frac{x^3+1}{6x^2+7}$.

#6. (6 pts) Find the equation of the tangent line to the curve $y = \sin\left(\frac{\pi x^2}{4}\right)$ at the point $(1, \frac{\sqrt{2}}{2})$.

#7. (7 pts) Given $y = \sqrt{x}$, use the definition of the derivative to compute y'.

#8. (6 pts) Consider $y^2 + xy + \frac{3}{y} = 4 + x^2$. Use implicit differentiation to find y'.

Multiple Choice Circle the best answer. No work needed. No partial credit available.

#9. (4 pts) Given that $f(x) = |x - 5|$, which of the following statements is true.

A. $f(x)$ is continuous and differentiable on $(-\infty, \infty)$.

B. $f(x)$ is continuous on $(-\infty, \infty)$ and differentiable on $(-\infty, 5) \cup (5, \infty)$.

C. $f(x)$ is continuous and differentiable on $(-\infty, 5) \cup (5, \infty)$.

D. $f(x)$ is differentiable on $(-\infty, \infty)$, but not continuous at $x = 5$.

E. $f(x)$ is not defined at $x = 5$.

#10. (4 pts) Suppose that $f(x)$ is continuous and differentiable and that $f'(x) > 0$ always and $f(0) = 3$. What is true about $f(1)$?

A. It is possible that $f(1) = 3$.

B. It must be that $f(1) < 3$.

C. It must be that $f(1) > 3$.

D. There is not enough information.

#11. (4 pts) Given that $x^2 + y^2 = 9$, which of the following is true?
(**Hint**: *Use implicit differentiation or sketch the graph.*)

A. $y' > 0$ always.

B. $y' < 0$ always.

C. $y' > 0$ in the I and III quadrants.

D. $y' > 0$ in the II and IV quadrants.

E. None of the above.

#12. (12 pts) Suppose the height of an object is modeled by $h(t) = 10t - 2t^2$ m, with time measured in seconds.

(a) (4 pts) When does the object reach its maximum height?

A. 2.5s

B. 25s

C. 10s

D. 4s.

E. None of the above.

(b) (4 pts) What is the maximum height of the object?

A. 25m

B. 12.5m

C. 50m

D. 2.5m

E. None of the above.

(c) (4 pts) What is the direction of the object at the time $t = 4$s

A. downward

B. upward

C. There is not enough information.

#13. (4 pts) Use the squeeze theorem to evaluate $\lim_{x\to 0} \sqrt{\frac{x^3+x^2}{\pi}} \sin\frac{\pi}{x}$.

A. 1

B. 0

C. DNE

D. $\frac{1}{\pi}$

#14. (4 pts) Calculate the derivative of $f(x) = \cos(\tan x)$.

A. $f'(x) = -\sin(\sec^2 x)$

B. $f'(x) = \sin(\tan x)\sec^2 x$

C. $f'(x) = -\sin(\tan x)\sec^2 x$

D. $f'(x) = \cos(\sec^2 x)$

E. $f'(x) = -\sin x \sec^2 x$

#15. (4 pts) The velocity of a particle moving back and forth along a straight line is given by $v(t) = 2\sin(\pi t) + 3\cos(\pi t)$, where time is measured in seconds.
What does $v'(t) = 2\pi\cos(\pi t) - 3\pi\sin(\pi t)$ represent?

A. The average rate of change of the position of the particle over any 1-second interval.

B. The instantaneous rate of change of the velocity.

C. The speed at which the particle is moving.

D. The average rate of change of the velocity of the particle over any 1-second interval.

E. The instantaneous rate of change of the position.

More Challenging Question(s). Error Carried Forward.

#16. (14 pts) Newton's Law of Gravitation says the magnitude of the force, F, exerted by a body of mass m on the body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies.

(a) (4 pts) Calculate $\frac{\mathrm{d}F}{\mathrm{d}r}$.

(b) (2 pts) Explain the physical meaning of $\frac{\mathrm{d}F}{\mathrm{d}r}$.

(c) (4 pts) Suppose the earth attracts an object with a force that decreases at a rate of 2N/km when $r = 20,000$km. How is this force changing when $r = 10,000$km? (*Include units*).