Standard Response Questions. Show all your work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

#1. (4 pts) Find $f'(x)$ if $f(x) = x^3 \cos(x)$.

Solution:

$$f'(x) = (x^3 \cos x)' = (x^3)' \cos x + x^3 \cos' x = \boxed{3x^2 \cos x - x^3 \sin x}$$

#2. (4 pts) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \sqrt{x^3 + 3x}$.

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\sqrt{x^3+3x}\right)' = \left((x^3+3x)^{1/2}\right)' = \frac{1}{2}(x^3+3x)^{-1/2}(x^3+3x)' = \boxed{\frac{3x^2+3}{2\sqrt{x^3+3x}}}$$

#3. (4 pts) Find **the equation** of the line tangent to $y = \frac{2x-3}{x^2+1}$ through $(1, -\frac{1}{2})$.

Solution:

$$y' = \left(\frac{2x-3}{x^2+1}\right)' = \frac{(2x-3)'(x^2+1) - (2x-3)(x^2+1)'}{(x^2+1)^2} = \frac{2(x^2+1) - (2x-3)2x}{(x^2+1)^2}$$

At $x = 1$ we get

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 \cdot 2 - (-1) \cdot 2}{2^2} = \frac{6}{4} = \frac{3}{2}$$

So the slope of the tangent line through $(1, -\frac{1}{2})$ is $\frac{3}{2}$. Hence the equation of the tangent line is

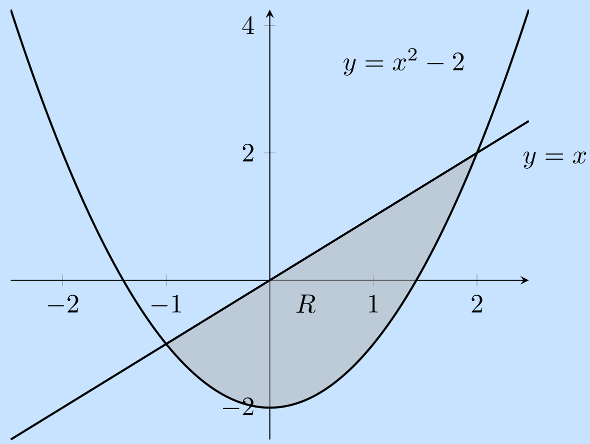
$$y - (-\frac{1}{2}) = \frac{3}{2}(x-1)$$

$$\boxed{y + \frac{1}{2} = \frac{3}{2}x - \frac{3}{2}}$$

$$\boxed{y = \frac{3}{2}x - 2}$$

#4. (6 pts) Sketch the bounded region R between the curves $y = x^2 - 2$ and $y = x$. Then find the area of R. (*You do not need to simplify your answer).*

Solution:



To compute the area of the region R we first need to find the intersection points:

$$
\begin{aligned}
x^2 - 2 &= x \\
x^2 - x - 2 &= 0 \\
(x+1)(x-2) &= 0 \\
x = -1 \quad \text{or} \quad x &= 2
\end{aligned}
$$

From the sketch we see that on the interval $[-1, 2]$ the larger function is $y = x$. So the area of R is

$$
\begin{aligned}
\int_{-1}^{2} x - (x^2 - 2)\,\mathrm{d}x &= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\right]_{-1}^{2} \\
&= \boxed{\left(\frac{1}{2}2^2 - \frac{1}{3}2^3 + 2\cdot 2\right) - \left(\frac{1}{2}(-1)^2 - \frac{1}{3}(-1)^3 + 2\cdot(-1)\right)} \\
&= 2 - \frac{8}{3} + 4 - \frac{1}{2} - \frac{1}{3} + 2 = 8 - \frac{8+1}{3} - \frac{1}{2} = 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{10-1}{2} = \boxed{\frac{9}{2}}
\end{aligned}
$$

#5. (6 pts) Use the Intermediate Value Theorem to show that there is at least one solution to the equation

$$\sqrt[3]{x} + 4x = 12.$$

(**Note:** *You need to justify why the IVT can be applied.*)

Solution: Let $f(x) = \sqrt[3]{x} + 4x$ and note that f is continuous everywhere. Hence we can apply the Intermediate Value Theorem as long as we can find numbers a and b such that 12 lies between $f(a)$ and $f(b)$. We have

$$
\begin{aligned}
f(1) &= \sqrt[3]{1} + 4\cdot 1 = 1 + 4 \;\; = 5 \;\; < 12 \\
f(8) &= \sqrt[3]{8} + 4\cdot 8 = 2 + 32 = 34 > 12
\end{aligned}
$$

Hence 12 is between $f(1)$ and $f(8)$ and so the IVT show that there exists c in $(1, 8)$ with $f(c) = 12$. Then c

is a solution to $\sqrt[3]{x}+4x=12$.

Remark: I chose 1 and 8 as my numbers since its easy to compute $\sqrt[3]{1}$ and $\sqrt[3]{8}$. One could have also used 2 and 3:

$$\begin{aligned} f(2) &= \sqrt[3]{2}+4\cdot 2 < 2+8 \;\; = 10 < 12 \\ f(3) &= \sqrt[3]{4}+4\cdot 3 > 1+12 = 13 > 12 \end{aligned}$$

The computation is slighly more complicated, but one gets a better result: We now know that the equation has a solution in the interval $(2,3)$.

#6. (8 pts) Use the definition of the derivative (as a limit) to calculated $f'(x)$ for $f(x)=2x^2+3$.
(*There will be no credit for other methods.*)

Solution:

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{(2(x+h)^2+3)-(2x^2+3)}{h} \\ &= \lim_{h\to 0}\frac{(2(x^2+2xh+h^2)+3)-(2x^2+3)}{h} \\ &= \lim_{h\to 0}\frac{\cancel{2x^2}+4xh+2h^2+\cancel{3}-\cancel{2x^2}-\cancel{3}}{h} \\ &= \lim_{h\to 0}\frac{4xh+2h^2}{h} \\ &= \lim_{h\to 0}\frac{(4x+h)h}{h} \\ &= \lim_{h\to 0} 4x+h \\ &= 4x+0 \\ &\boxed{= 4x} \end{aligned}$$

#7. (4 pts) Solve the initial value problem: $\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{x}$, $y(9)=0$.

Solution: y is a antiderivative of $\sqrt{x}$. Since $\sqrt{x}=x^{1/2}$ we conclude that

$$y=\frac{2}{3}x^{3/2}+C=\frac{2}{3}\sqrt{x}^3+C$$

where C is a constant. Thus

$$0=y(9)=\frac{2}{3}\sqrt{9}^3+C=\frac{2}{3}3^3+C=2\cdot 3^2+C=18+C$$

and so $C=-18$. Thus

$$\boxed{y=\frac{2}{3}\sqrt{x}^3-18}$$

#8. (8 pts) The height and base of a triangle are changing with time. The height is increasing at a rate of $3\,\frac{\mathrm{cm}}{\mathrm{min}}$, while the area is changing at a rate of $11\,\frac{\mathrm{cm}^2}{\mathrm{min}}$. At what rate is the length of the base changing when the height is $5\,\mathrm{cm}$ and the area is $10\,\mathrm{cm}^2$?

Solution: Let A, h and b be the area, height and base of triangle, respectively. Then

$$A = \frac{1}{2}hb$$

We know that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 3 \quad \text{and} \quad \frac{\mathrm{d}A}{\mathrm{d}t} = 11$$

and need to compute $\frac{\mathrm{d}b}{\mathrm{d}t}$ when $h = 5$ and $A = 10$.

Differentiating both sides of $A = \frac{1}{2}hb$ with respect to t gives

$$\begin{aligned} \frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{1}{2}\left(\frac{\mathrm{d}h}{\mathrm{d}t}b + h\frac{\mathrm{d}b}{\mathrm{d}t}\right) \\ 2\frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{\mathrm{d}h}{\mathrm{d}t}b + h\frac{\mathrm{d}b}{\mathrm{d}t} \end{aligned}$$

Suppose now that $h = 5$ and $A = 10$. Using $A = \frac{1}{2}hb$ we get

$$b = \frac{2A}{h} = \frac{2 \cdot 10}{5} = 4$$

Recall that $\frac{\mathrm{d}A}{\mathrm{d}t} = 11$ and $\frac{\mathrm{d}h}{\mathrm{d}t} = 3$. So

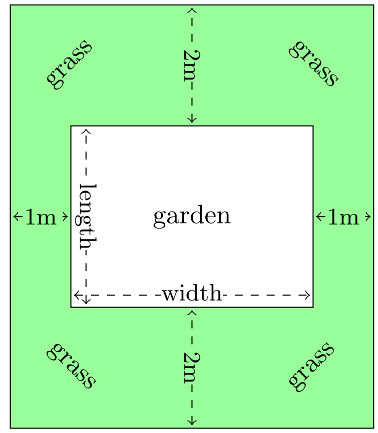
$$\begin{aligned} 2\frac{\mathrm{d}A}{\mathrm{d}t} &= \frac{\mathrm{d}h}{\mathrm{d}t}b + h\frac{\mathrm{d}b}{\mathrm{d}t} \\ 2 \cdot 11 &= 3 \cdot 4 + 5\frac{\mathrm{d}b}{\mathrm{d}t} \\ 22 &= 12 + 5\frac{\mathrm{d}b}{\mathrm{d}t} \\ \frac{\mathrm{d}b}{\mathrm{d}t} &= \frac{22 - 12}{5} = \frac{10}{5} = \boxed{2\frac{\mathrm{cm}}{\mathrm{min}}} \end{aligned}$$

#9. (4 pts) Evaluate $\int \sin^7(x)\cos(x)\,\mathrm{d}x$.

Solution:

$$\begin{aligned} u &= \sin(x) \\ \mathrm{d}u &= \cos(x)\,\mathrm{d}x \\ \int \sin^7(x)\cos(x)\,\mathrm{d}x &= \int u^7\,\mathrm{d}u = \frac{1}{8}u^8 + C = \frac{1}{8}\sin^8(x) + C \end{aligned}$$

#10. (12 pts) A man wants to build a rectangular garden with an area of $18\,\mathrm{m}^2$ with a grass border 1 meter wide on two sides and 2 meters wide on the other two sides (see picture). Find the length and width of the garden that minimizes the total area (i.e the area of the garden and the grass).
Use techniques of calculus to justify that your answer is a minimum.



Solution: Let l and w be the length and width of the garden, and let A be the total area.
The garden is supposed to have an area of $18\,\mathrm{m}^2$. So

$$18 = lw$$

and

$$w = \frac{18}{l}$$

Note that l cannot be negative, so the domain for l is $(0, \infty)$.
The garden together with the grass forms rectangle of width $w + 2 \cdot 1$ and height $l + 2 \cdot 2$. Thus

$$A = (w + 2)(l + 4) = wl + 2l + 4w + 8$$

and since $w = \frac{18}{l}$:

$$A = \frac{18}{l} l + 2l + 4\frac{18}{l} + 8 = 18 + 2l + \frac{72}{l} + 8 = 2l + \frac{72}{l} + 26$$

So

$$\frac{\mathrm{d}A}{\mathrm{d}l} = 2 - \frac{72}{l^2} = \frac{2l^2 - 72}{l^2} = 2\frac{l^2 - 36}{l^2}$$

Hence $\frac{\mathrm{d}A}{\mathrm{d}l}$ is defined for all l in the domain $(0, \infty)$ and $\frac{\mathrm{d}A}{\mathrm{d}l} = 0$ at $l = \sqrt{36} = 6$. (Note that $-\sqrt{36}$ is not in the domain.) We have

	$(0, 6)$	$(6, \infty)$
$l^2 - 36$	$-$	$+$
l^2	$+$	$+$
f'	$-$	$+$
f	$\searrow$	$\nearrow$

So the $\boxed{\text{First Derivative Test for Absolute Extrema}}$ shows that A has absolute minimum at $l = \boxed{6\,\mathrm{m}}$. The width when $l = 6$ is $w = \frac{18}{l} = \frac{18}{6} = \boxed{3\,\mathrm{m}}$.

#11. (12 pts) Consider the function and its derivatives given by

$$f(x) = \frac{x^2 + 1}{x} \qquad f'(x) = 1 - \frac{1}{x^2} \qquad f''(x) = \frac{2}{x^3}$$

(a) (2 pts) Find all vertical, horizontal, and slant asymptotes of $y = f(x)$ (if they exist).

Solution: Note that the denominator of $f(x)$ is zero at $x = 0$, but the numerator is not. Thus

$$\boxed{x = 0 \text{ is a vertical asymptote of } f}$$

We have

$$f(x) = \frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + \frac{1}{x}$$

and so

$$\boxed{y = x \text{ is a slant asymptote of } f}$$

We have

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x + \frac{1}{x} = \pm\infty$$

and so

$$\boxed{f \text{ does not have a horizontal asymptote}}$$

(b) (4 pts) Find the interval(s) where f is increasing and where f is decreasing.

Express your answers using interval notation.

Solution:

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$$

Thus f' is not defined at $x = 0$ and $f'(x) = 0$ and $x = \pm\sqrt{1} = \pm 1$. Also

	$(\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$x^2 - 1$	$+$	$-$	$-$	$+$
x^2	$+$	$+$	$+$	$+$
f'	$+$	$-$	$-$	$+$
f	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

Since f is continuous at $x = \pm 1$ and is not defined at $x = 0$ we conclude:

$$\boxed{f \text{ is increasing on } (-\infty, -1] \text{ and on } [1, \infty)}$$

and

$$\boxed{f \text{ is decreasing on } [-1, 0) \text{ and on } (0, 1]}$$

(c) (2 pts) Find the interval(s) where f is concave up and where f is concave down.

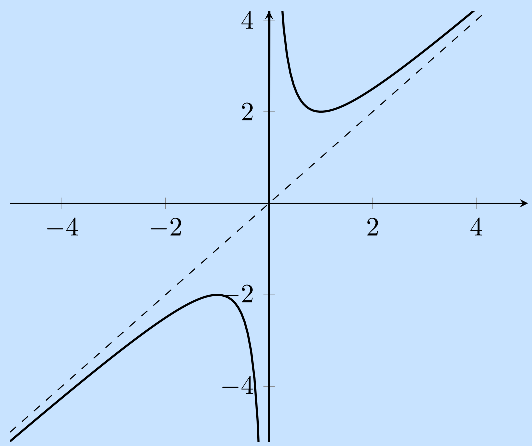
Express your answers using interval notation.

Solution: Since $f''(x) = \frac{2}{x^3}$ we see that $f''(x) > 0$ for $x > 0$ and $f''(x) < 0$ for $x < 0$. Thus

$$\boxed{f \text{ is concave up on } (0, \infty) \text{ and concave down on } (-\infty, 0)}$$

(d) (4 pts) Using the information in parts (a)-(c), sketch the graph of $y = f(x)$.

Solution:



Multiple Choice. Circle the best answer. No work needed.
No partial credit available. No credit will be given for choices not clearly marked.

#12. (3 pts) Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x \, \mathrm{d}x$.

A. $\frac{2\sqrt{2}}{3}$ B. $\boxed{1}$ C. $\frac{2\sqrt{2}}{3} - \frac{1}{3}$ D. $\frac{\sqrt{2}}{3} - \frac{1}{3}$ E. 0

Solution:

$$\int_0^{\frac{\pi}{4}} \sec^2 x \, \mathrm{d}x = [\tan x]_0^{\frac{\pi}{4}} = \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 1.$$

So B. is the correct answer.

#13. (3 pts) Evaluate $\lim_{x \to 0} \dfrac{\sin(x)}{\cos(3x)}$.

A. $\frac{1}{3}$ B. $\boxed{0}$ C. 1 D. 3 E. -1

Solution: Note that $\cos(3 \cdot 0) = \cos(0) = 1 \neq 0$. Thus $\frac{\sin(x)}{\cos(3x)}$ is continuous at $x = 0$ and so

$$\lim_{x \to 0} \frac{\sin(x)}{\cos(3x)} = \frac{\sin(0)}{\cos(3 \cdot 0)} = \frac{0}{1} = 0$$

So B. is the correct answer.

#14. (3 pts) Find $\frac{dy}{dx}$ at $(1,2)$ if x and y satisfy the implicit equation $3x^2 + y^3 = 11$.

A. -1 B. $\boxed{-\frac{1}{2}}$ C. 2 D. 1 E. 3

Solution: Differentiating both sides of the equation

$$3x^2 + y^3 = 11$$

with respect to x gives

$$6x + 3y^2 y' = 0$$
$$y' = \frac{-6x}{3y^2} = -\frac{2x}{y^2}$$

For $x = 1$ and $y = 2$ we get

$$y' = -\frac{2x}{y^2} = -\frac{2 \cdot 1}{2^2} = -\frac{1}{2}$$

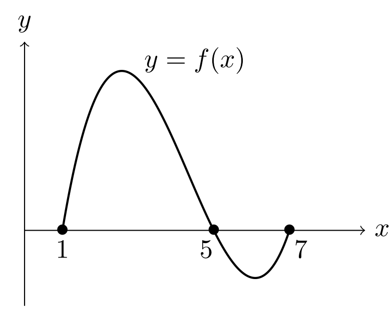
So B. is the correct answer.

#15. (3 pts) The function $y = f(x)$ (see graph, right) satisfies

$$\int_1^7 f(x)\,dx = 8, \quad \int_5^7 f(x)\,dx = -3,$$

The average value of $f(x)$ on the interval $[1,5]$ is

A. 11 B. $\boxed{\frac{11}{4}}$ C. $\frac{11}{6}$ D. $\frac{5}{11}$ E. 5



Solution: We have

$$8 = \int_1^7 f(x)\,dx = \int_1^5 f(x)\,dx \ + \ \int_5^7 f(x)\,dx = \int_1^5 f(x)\,dx \ + \ (-3)$$

and so

$$\int_1^5 f(x)\,dx = 8 + 3 = 11$$

Thus

$$\mathrm{av}_1^5(f) = \frac{1}{5-1}\int_1^5 f(x)\,dx \ = \ \frac{1}{4} \cdot 11 = \frac{11}{4}$$

Thus B. is the correct answer.

#16. (3 pts) The horizontal and vertical asymptotes of the graph of $y = \frac{2x+1}{x-1}$ are

$\boxed{\text{A. } y = 2,\, x = 1}$, B. $y = -\frac{1}{2},\, x = -1$, C. $y = 1,\, x = 1$, D. $y = -1,\, x = -1$, E. None of the above.

Solution: The denominator of $\frac{2x+1}{x-1}$ is zero at $x = 1$, but the numerator is not. Thus $x = 1$ is a vertical asymptote. Also

$$\lim_{x\to\pm\infty} \frac{2x+1}{x-1} = \lim_{x\to\infty} \frac{x(2+\frac{1}{x})}{x(1-\frac{1}{x})} = \lim_{x\to\infty} \frac{2+\frac{1}{x}}{1-\frac{1}{x}} = \frac{2+0}{1+0} = 2.$$

Hence $y = 2$ is a horizontal asymptote. Thus A. is the correct answer.

#17. (3 pts) Evaluate $\int_{-1}^{1} t^3\sqrt{1+t^4}\,\mathrm{d}t$.

A. $\frac{2\sqrt{2}}{3}$ B. $-\frac{2\sqrt{2}}{3}$ $\boxed{\text{C. } 0}$ D. $\frac{\sqrt{2}}{2}$ E. $-\frac{\sqrt{2}}{2}$

Solution 1: Let $f(t) = t^3\sqrt{1+t^4}$. Then

$$f(-t) = (-t)^3\sqrt{1+(-t)^4} = -t^3\sqrt{1+t^4} = -f(t)$$

Hence f is an odd function and so

$$\int_{-1}^{1} t^3\sqrt{1+t^4}\,\mathrm{d}t = 0$$

Thus C. is the correct answer.

Solution 2:

$$\begin{gathered}
u = 1+t^4 \\
\mathrm{d}u = 4t^3\,\mathrm{d}t \\
\frac{1}{4}\,\mathrm{d}u = t^3\,\mathrm{d}t \\
t = 1: \quad u = 1+1^4 = 2 \\
t = -1: \quad u = 1+(-1)^4 = 2 \\
\int_{-1}^{1} t^3\sqrt{1+t^4}\,\mathrm{d}t = \int_{-1}^{1} \sqrt{1+t^4}t^3\,\mathrm{d}t = \int_{2}^{2} \sqrt{u}\frac{1}{4}\,\mathrm{d}u = 0
\end{gathered}$$

Thus C. is the correct answer.

#18. (3 pts) Consider the function $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$. By differentiating (you don't have to do this!), one finds that

$$f'(x) = 12(x+1)(x-1)^2 \quad \text{and} \quad f''(x) = 12(x-1)(3x+1)$$

Using these formulas...

I. $x = -1$ is a critical point.

II. $x = -1$ is a local maximum.

III. $x = -1$ is a local minimum.

IV. $x = -1$ is an inflection point.

A. Only I. is true.

B. Only I. and II. are true.

C. $\boxed{\text{Only I. and III. are true.}}$

D. Only I., II. and IV. are true.

E. Only I., II. and IV. are true.

Solution: From $f'(x) = 12(x+1)(x-1)^2$ we see that $f'(-1) = 0$. Thus $x = -1$ is a critical number. Also since $f''(x) = 12(x-1)(3x+1)$:

$$f''(-1) = 12 \cdot (-1-1) \cdot (3 \cdot (-1) + 1) = 12 \cdot (-2) \cdot (-2) = 48 > 0$$

Hence $x = -1$ is not an inflection point, and f is concave up at $x = -1$. The Second Derivative Test now shows that $x = -1$ is a local minimum. Thus I. and III. are true, and II. and IV are false. Hence C. is the correct answer.

#19. (3 pts) Find $F'(1)$ for $F(x) = \int_0^{x^2} \frac{dt}{t^2+3}$.

A. $\frac{1}{4}$ $\quad$ $\boxed{\text{B. } \frac{1}{2}}$ $\quad$ C. $\frac{1}{8}$ $\quad$ D. 1 $\quad$ E. 2

Solution: Put $g(x) = \frac{1}{x^2+3}$ and $G(x) = \int_0^x g(t)\,dt = \int_0^x \frac{dt}{t^2+3}$. Then $F(x) = G(x^2)$ and by the FToC, $G'(x) = g(x)$. Thus

$$F'(x) = \big(G(x^2)\big)' = G'(x^2)(x^2)' = g(x^2)2x = \frac{1}{(x^2)^2+3}2x = \frac{2x}{x^4+3}$$

and so

$$F'(1) = \frac{2 \cdot 1}{1^4+3} = \frac{2}{4} = \frac{1}{2}$$

Thus B. is the correct answer.

#20. (3 pts) Evaluate $\lim\limits_{x \to 0^-} \dfrac{x^2+5x-10}{x^2-3x}$.

A. -10 $\quad$ B. $-\frac{10}{3}$ $\quad$ C. 1 $\quad$ $\boxed{\text{D. } -\infty}$ $\quad$ E. ∞

Solution:

$$\begin{aligned}\lim_{x \to 0^-} \frac{x^2+5x-10}{x^2-3x} &= \lim_{x \to 0^-} \frac{x^2+5x-10}{x(x-3)} \\ &\left(\frac{\text{close to } -10}{\text{very small negative number} \times \text{close to } -3} = \frac{\text{negative number}}{\text{very small positive number}}\right) \\ &= -\infty\end{aligned}$$

So D. is the correct answer.

#21. (3 pts) One can approximate a solution to $x^4 = 14$ using Newton's Method. If one starts with $x_1 = 2$, then x_2 is which of the following?

A. 0 $\qquad$ $\boxed{\text{B. } \frac{31}{6}}$ $\qquad$ C. $\frac{33}{16}$ $\qquad$ D. $\frac{7}{4}$ $\qquad$ E. $\frac{9}{4}$

Solution: Let $f(x) = x^4 - 14$. Then

$$\begin{aligned} f'(x) &= 4x^3 \\ f(x_1) &= f(2) = 2^4 - 14 = 2 \\ f'(x_1) &= f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32 \\ x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{2}{32} = 2 - \frac{1}{16} = \frac{32-1}{16} = \frac{31}{16} \end{aligned}$$

So B. is the correct answer.

#22. (3 pts) Evaluate $\lim_{x\to\infty} \sqrt{\dfrac{9x^2+5x}{x^2-2}}$.

A. 0 $\qquad$ B. 1 $\qquad$ $\boxed{\text{C. } 3}$ $\qquad$ D. 9 $\qquad$ E. ∞

Solution:

$$\lim_{x\to\infty} \sqrt{\frac{9x^2+5x}{x^2-2}} = \lim_{x\to\infty} \sqrt{\frac{x^2(9+\frac{5}{x})}{x^2(1-\frac{2}{x^2})}} = \lim_{x\to\infty} \sqrt{\frac{9+\frac{5}{x}}{1-\frac{2}{x^2}}} = \sqrt{\frac{9+0}{1-0}} = \sqrt{9} = 3$$

Thus C. is the correct answer.

#23. (3 pts) For what value of c is $\int_2^{10} (x+c)\,\mathrm{d}x = 0$?

A. 0 $\qquad$ B. -1 $\qquad$ C. 5 $\qquad$ $\boxed{\text{D. } -6}$ $\qquad$ E. 3

Solution:

$$\int_2^{10} x + c\,\mathrm{d}x = \left[\frac{1}{2}x^2 + cx\right]_2^{10} = \left(\frac{1}{2}10^2 + c \cdot 10\right) - \left(\frac{1}{2}2^2 + c \cdot 2\right) = 50 + 10c - 2 - 2c = 48 + 8c$$

So

$$\begin{aligned} \int_2^{10} (x+c)\,\mathrm{d}x &= 0 \\ 48 + 8c &= 0 \\ c &= -\frac{48}{8} = -6 \end{aligned}$$

Thus D. is the correct answer.