Standard Response Questions. Show all your work to receive credit. Please **BOX** your final answer.

#1. (4 pts) Find $f'(x)$ if $f(x) = x^3 \cos(x)$.

#2. (4 pts) Find $\frac{dy}{dx}$ if $y = \sqrt{x^3 + 3x}$.

#3. (4 pts) Find **the equation** of the line tangent to $y = \frac{2x-3}{x^2+1}$ through $(1, -\frac{1}{2})$.

#4. (6 pts) Sketch the bounded region R between the curves $y = x^2 - 2$ and $y = x$. Then find the area of R. (*You do not need to simplify your answer).*

#5. (6 pts) Use the Intermediate Value Theorem to show that there is at least one solution to the equation

$$\sqrt[3]{x} + 4x = 12.$$

(**Note:** *You need to justify why the IVT can be applied.*)

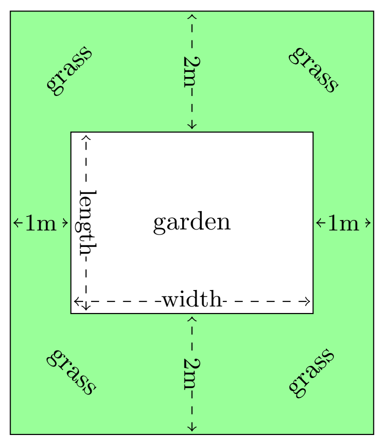
#6. (8 pts) Use the definition of the derivative (as a limit) to calculated $f'(x)$ for $f(x) = 2x^2 + 3$.
(*There will be no credit for other methods.*)

#7. (4 pts) Solve the initial value problem: $\frac{dy}{dx} = \sqrt{x}$, $y(9) = 0$.

#8. (8 pts) The height and base of a triangle are changing with time. The height is increasing at a rate of $3 \frac{\text{cm}}{\text{min}}$, while the area is changing at a rate of $11 \frac{\text{cm}^2}{\text{min}}$. At what rate is the length of the base changing when the height is $5\,\text{cm}$ and the area is $10\,\text{cm}^2$?

#9. (4 pts) Evaluate $\int \sin^7(x) \cos(x)\, dx$.

#10. (12 pts) A man wants to build a rectangular garden with an area of $18\,\text{m}^2$ with a grass border 1 meter wide on two sides and 2 meters wide on the other two sides (see picture). Find the length and width of the garden that minimizes the total area (i.e the area of the garden and the grass).
Use techniques of calculus to justify that your answer is a minimum.



#11. (12 pts) Consider the function and its derivatives given by

$$f(x) = \frac{x^2+1}{x} \qquad f'(x) = 1 - \frac{1}{x^2} \qquad f''(x) = \frac{2}{x^3}$$

(a) (2 pts) Find all vertical, horizontal, and slant asymptotes of $y = f(x)$ (if they exist).

(b) (4 pts) Find the interval(s) where f is increasing and where f is decreasing.
Express your answers using interval notation.

(c) (2 pts) Find the interval(s) where f is concave up and where f is concave down.
Express your answers using interval notation.

(d) (4 pts) Using the information in parts (a)-(c), sketch the graph of $y = f(x)$.

Multiple Choice. Circle the best answer. No work needed.
No partial credit available. No credit will be given for choices not clearly marked.

#12. (3 pts) Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x \, \mathrm{d}x$.

A. $\frac{2\sqrt{2}}{3}$ B. 1 C. $\frac{2\sqrt{2}}{3} - \frac{1}{3}$ D. $\frac{\sqrt{2}}{3} - \frac{1}{3}$ E. 0

#13. (3 pts) Evaluate $\displaystyle\lim_{x \to 0} \frac{\sin(x)}{\cos(3x)}$.

A. $\frac{1}{3}$ B. 0 C. 1 D. 3 E. -1

#14. (3 pts) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $(1, 2)$ if x and y satisfy the implicit equation $3x^2 + y^3 = 11$.

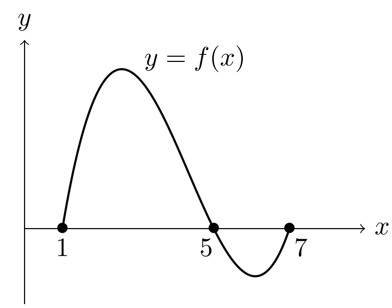
A. -1 B. $-\frac{1}{2}$ C. 2 D. 1 E. 3

#15. (3 pts) The function $y = f(x)$ (see graph, right) satisfies

$$\int_1^7 f(x) \, \mathrm{d}x = 8, \quad \int_5^7 f(x) \, \mathrm{d}x = -3,$$

The average value of $f(x)$ on the interval $[1, 5]$ is

A. 11 B. $\frac{11}{4}$ C. $\frac{11}{6}$ D. $\frac{5}{11}$ E. 5



#16. (3 pts) The horizontal and vertical asymptotes of the graph of $y = \frac{2x+1}{x-1}$ are

A. $y = 2$ and $x = 1$, B. $y = -\frac{1}{2}$ and $x = -1$, C. $y = 1$ and $x = 1$, D. $y = -1$ and $x = -1$, E. None of the above.

#17. (3 pts) Evaluate $\int_{-1}^{1} t^3 \sqrt{1 + t^4} \, \mathrm{d}t$.

A. $\frac{2\sqrt{2}}{3}$ B. $-\frac{2\sqrt{2}}{3}$ C. 0 D. $\frac{\sqrt{2}}{2}$ E. $-\frac{\sqrt{2}}{2}$

#18. (3 pts) Consider the function $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x + 1$. By differentiating (you don't have to do this!), one finds that

$$f'(x) = 12(x+1)(x-1)^2 \quad \text{and} \quad f''(x) = 12(x-1)(3x+1)$$

Using these formulas...

I. $x = -1$ is a critical point.

II. $x = -1$ is a local maximum.

III. $x = -1$ is a local minimum.

IV. $x = -1$ is an inflection point.

A. Only I. is true.

B. Only I. and II. are true.

C. Only I. and III. are true.

D. Only I., II. and IV. are true.

E. Only I., II. and IV. are true.

#19. (3 pts) Find $F'(1)$ for $F(x) = \int_0^{x^2} \frac{\mathrm{d}t}{t^2+3}$.

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{1}{8}$ D. 1 E. 2

#20. (3 pts) Evaluate $\lim_{x \to 0^-} \frac{x^2 + 5x - 10}{x^2 - 3x}$.

A. -10 B. $-\frac{10}{3}$ C. 1 D. $-\infty$ E. ∞

#21. (3 pts) One can approximate a solution to $x^4 = 14$ using Newton's Method. If one starts with $x_1 = 2$, then x_2 is which of the following?

A. 0 B. $\frac{31}{6}$ C. $\frac{33}{16}$ D. $\frac{7}{4}$ E. $\frac{9}{4}$

#22. (3 pts) Evaluate $\lim_{x \to \infty} \sqrt{\frac{9x^2 + 5x}{x^2 - 2}}$.

A. 0 B. 1 C. 3 D. 9 E. ∞

#23. (3 pts) For what value of c is $\int_2^{10} (x + c)\,\mathrm{d}x = 0$?

A. 0 B. -1 C. 5 D. -6 E. 3