MTH 132 **Exam 2** **Fall 2016**

Standard Response Questions. Show all your work to receive credit. Please **BOX** your final answer.

#1. (18 pts)

(a) (6 pts) Find the most general antiderivative of $f(x) = 4\cos x + 8$.

(b) (6 pts) Evaluate $\int_1^2 \frac{5-7x^6}{x^4}\,\mathrm{d}x$.

(c) (6 pts) Let $F(x) = \int_{x^3}^1 \frac{1}{t^2+1}\,\mathrm{d}x$. Find $F'(x)$.

#2. (18 pts)

(a) (8 pts) A particle is moving along a line with acceleration (in $\mathrm{m/s^2}$) given by $a(t) = 4t^3 + 2\sin t$. Given that the initial velocity is $v(0) = 5\,\mathrm{m/s}$, find the velocity at the time $t = \pi$ seconds.

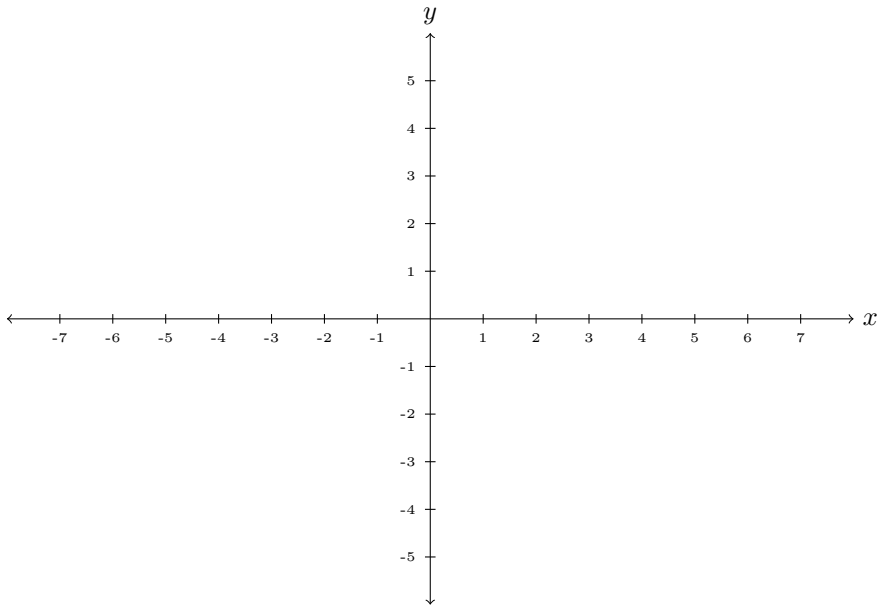
(b) (10 pts) Use a linearization to find a good approximation of $\sqrt{9.01}$.

#3. (18 pts)

(a) (10 pts) There are many curves $y = f(x)$ which satisfy the following conditions:

- f is continuous and the curve $y = f(x)$ has a slant (or oblique) asymptote given by $y = x$.
- $f'(x) > 0$ for $x \in (-\infty, -1) \cup (3, \infty)$ and $f'(x) < 0$ for $x \in (-1, 3)$.
- $f''(x) > 0$ for $x \in (-\infty, -3) \cup (1, \infty)$ and $f''(x) < 0$ for $x \in (-3, 1)$.

Sketch the graph of one such curve below, making sure that all the above conditions are demonstrated by the curve you draw. Identify with a large dot • the locations of any local max/min or points of inflections on your graph, and give the x-values of these points in the boxes below.



Fill in this information:

f has local minimums at these x-values:
f has local maximums at these x-values:
f has inflection points at these x-values:

(b) (8 pts) Find the critical numbers (i.e., critical points) of the function $f(x) = x^{3/2} + \frac{6}{\sqrt{x}}$.

#4. (18 pts) Suppose $f(x) = \frac{x}{x^2+1}$, $\quad f'(x) = \frac{1-x^2}{(x^2+1)^2}$, $\quad f''(x) = \frac{2(x^3-3x)}{(x^2+1)^3}$.

Answer the following questions or enter **none** *in the case of no answer.*

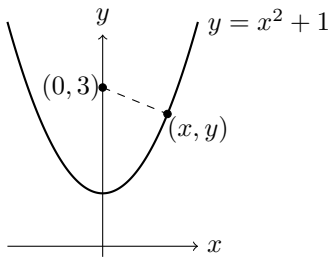
(a) (4 pts) Does f have symmetry about the y-axis (even function), symmetry about the origin (odd function), both, or neither? Justify your answer.

(b) (7 pts) Find the largest interval(s) where f is increasing and the largest interval(s) where f is decreasing. Express you answers using interval notation.

(c) (7 pts) Find the largest interval(s) where f is concave up and the largest interval(s) where f is concave down. Express you answers using interval notation.

#5. (18 pts)

(a) (8 pts) Consider the problem of finding the point (x, y) that lies on the curve $y = x^2 + 1$ in the first quadrant and which is closest to the point $(0, 3)$.
Define a function f of x which, if minimized, will give the x-coordinate of the point on the curve closest to $(0, 3)$. Also give the domain of this function which is *appropriate for the minimization problem.*



(b) (10 pts) Find the absolute maximum and the absolute minimum values of

$$f(x) = x^2(2x - 8)$$

on the interval $[-1, 2]$.

Multiple Choice. Circle the best answer. No work needed.
No partial credit available. No credit will be given for choices not clearly marked.

#6. (7 pts) If the Mean Value Theorem is applied to the function $f(x) = x^2 - 2x$ on the interval $[1, 4]$ which of the following values of c satisfy the conclusion of the Mean Value Theorem in this case?

A. $c = 1$ B. $c = \frac{3}{2}$ C. $c = 2$ D. $c = \frac{5}{2}$ E. $c = 3$

#7. (7 pts) Using three equally-spaced rectangles of equal width, find the upper sum approximation of the area between the curve $y = x^2$ and the x-axis from $x = -2$ to $x = 4$.

A. 8 B. 16 C. 24 D. 40 E. 48

#8. (7 pts) Evaluate $\int_0^3 \sqrt{9 - x^2}$. (*Hint: A definite integral represents an area.*)

A. $\frac{3\pi}{4}$ B. $\frac{3\pi}{2}$ C. $\frac{9\pi}{4}$ D. $\frac{9\pi}{2}$ E. 9π

#9. (7 pts) Which of the following is the equation for the horizontal asymptote for the curve $y = \frac{9x-2}{5-2x}$?

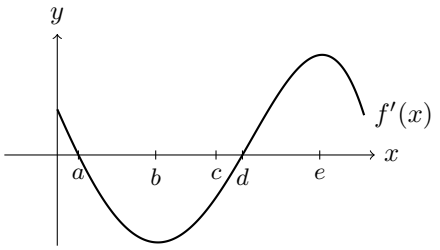
A. $y = \frac{9}{2}$ B. $y = -\frac{9}{2}$ C. $y = 0$ D. $y = -\frac{5}{2}$ E. $y = \frac{5}{2}$

#10. (7 pts) $\int_0^4 |3 - x| \, dx =$?

A. 6 B. 5 C. 4 D. 3 E. 2

#11. (7 pts) The graph of the first derivative $f'(x)$ of a function $f(x)$ is shown. At what value of x does f have a local maximum?

A. $x = a$ B. $x = b$ C. $x = c$ D. $x = d$ E. $x = e$

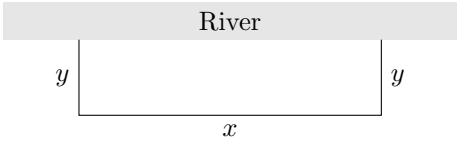


#12. (7 pts) Which of the following definite integrals is equivalent to the following limit of Riemann sums?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{8 + \frac{5i}{n}} \cdot \frac{5}{n}$$

A. $\int_8^{13} \sqrt{8 + 5x} \, dx$ B. $\int_0^5 \sqrt{8 + x} \, dx$ C. $\int_0^1 \sqrt{8 + 5x} \, dx$ D. $\int_0^5 5\sqrt{8 + x} \, dx$ E. None of the above

#13. (7 pts) A farmer wants to build a rectangular pen which will be bounded on one side by a river and on the other three sides by a wire fence. If the farmer has 60 m of wire to use, what is the largest area the farmer can enclose.



A. $200\,\text{m}^2$ B. $400\,\text{m}^2$ C. $450\,\text{m}^2$ D. $600\,\text{m}^2$ E. $900\,\text{m}^2$

#14. (7 pts) Suppose $\int_2^5 f(x) \, dx = 3$ and $\int_2^3 f(x) \, dx = -4$. Find $\int_3^5 2f(x) \, dx$.

A. -1 B. 7 C. -2 D. 14 E. 28