

Standard Response Questions. Show all your work to receive credit. Please **BOX** your final answer.

#1. (18 pts)

- (a) (10 pts) Consider the curve $y^2 + xy + x^3 = 3$. Find the slope of the curve at the point $(1, -2)$.
- (b) (8 pts) If $f(x) = \sec(\sin(x^2 + x))$, what is $f'(x)$? (*Do not simplify your answer!*).

#2. (18 pts) Consider $f(x) = \sqrt{1 - 2x}$.

- (a) (12 pts) Use the definition of the derivative to find $f'(x)$.
- (b) (6 pts) Use part (a) to find an equation of the tangent line of $f(x)$ at $x = -4$.

#3. (18 pts) A filter filled with liquid is in the shape of a vertex-down cone with a height of 8 inches and a diameter of 6 inches at its open (upper) end. If liquid drips out of the bottom of the filter at the constant rate of $7 \text{ in}^3/\text{min}$, how fast is the level of the liquid dropping when the liquid is 5 inches deep?

#4. (18 pts) A particle moves according to the law of motion $s = t^3 - 6t^2 + 5t, t \geq 0$, where t is measured in seconds and s in feet.

- (a) (3 pts) Find the average velocity over the interval $[0, 2]$.
- (b) (4 pts) Find the velocity at the time t .
- (c) (3 pts) What is the acceleration after 6 seconds?
- (d) (3 pts) What is the speed of the particle when the acceleration is zero?
- (e) (5 pts) For $t \geq 0$, when is the particle moving in the positive direction?

#5. (18 pts)

- (a) (10 pts) Use the Intermediate Value Theorem to show that there is a solution to the equation $\cos x = \sqrt{x}$. (*Make sure to justify why you can apply the IVT.*)
- (b) (8 pts) Consider the function $f(x) = \frac{x+2}{\cos(x)}$.

Where is the function continuous on $[0, 2\pi]$? (*Express your answer in interval notation*)

Multiple Choice Circle the best answer. No work needed. No partial credit given.

#6. (7 pts) For which real number c does $\lim_{x \rightarrow 2} \frac{cx^2 + 4}{x - 2}$ exist and is finite?

- A. $c = -2$, B. $c = -1$, C. $c = 0$, D. $c = 1$ E. $c = 2$.

#7. (7 pts) Compute $\lim_{x \rightarrow -2^-} \frac{|x^2 - 4|}{x + 2}$.

- A. $-\infty$, B. -4 , C. 0 , D. 4 E. ∞ .

#8. (7 pts) Compute the limit: $\lim_{x \rightarrow -3^+} \frac{x - 2}{x^2(x + 3)}$.

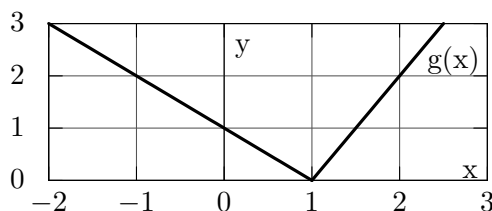
- A. $-\infty$, B. -3 , C. -1 , D. 1 E. ∞ .

#9. (7 pts) For what value of k will $f(x)$ be continuous for all values of x ?

$$f(x) = \begin{cases} \frac{x^2 - 3k}{x - 3} & \text{if } x \leq 2 \\ 8x - k & \text{if } x > 2 \end{cases}$$

- A. $k = 2$, B. $k = 3$, C. $k = 4$, D. $k = 5$ E. No value of k .

#10. (7 pts) Given the graph $y = g(x)$ below, find the limit $\lim_{h \rightarrow 0} \frac{g(1+h) - g(1)}{h}$.



- A. 0 , B. 1 , C. 2 , D. -1 E. Does not exist.

#11. (7 pts) Let $h(x) = \frac{2G(x)}{1+F(x)}$. Calculate $h'(2)$ if $F(2) = -3$, $G(2) = 5$, $F'(2) = 2$ and $G'(2) = 6$.

- A. 6 , B. 4 , C. $44/9$, D. 22 E. -11 .

#12. (7 pts) If $T(x) = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$, then $T'(x) =$

- A. $x + \frac{1}{x^2}$, B. $\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{3}{2}}}$, C. $\frac{1}{x^{\frac{1}{2}}} + \frac{1}{4x^{\frac{3}{2}}}$, D. $\frac{4x-1}{4x^{\frac{3}{2}}}$ E. $\frac{4}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{3}{2}}}$

#13. (7 pts) Find y'' if $y = \sin(x^2)$.

- A. $2 \cos(x^2) - 4x^2 \sin(x^2)$, B. $\cos(x^2) - \sin(x^2)$ C. $2x \cos(x^2) - 4x^2 \sin(x^2)$
D. $2x \cos(x^2) + 2x \sin(x^2)$, E. $-\sin(x^2)$.

#14. (7 pts) Find the limit $\lim_{x \rightarrow 0} \frac{\sin(x^2 + 6x)}{x}$.

- A. 0 , B. 1 , C. -1 , D. 6 E. Does not exist.