MTH 132-6 **Final Exam** **S08**

INSTRUCTIONS: Present your solution to each problem in a clear and orderly fashion. You must show your work. An answer alone without supporting work will not receive full credit.

Calculators are not allowed on this exam.

#1. (24 pts) Find the following limits:

(a) (8 pts) $\lim_{x\to 0} \frac{\sin 2x}{x} + \lim_{x\to\frac{\pi}{4}} \frac{\sin 2x}{x}$

(b) (8 pts) $\lim_{x\to 1^-} (8x+3)\frac{|x-1|}{x-1}$.

(c) (8 pts) $\lim_{x\to 0} \frac{x(\cos x - 1)}{\sin x - x}$ (**Ignore this part. It requires L'Hospital's Rule, which was not covered this year.**)

#2. (24 pts) Find the derivative of the following functions (**do not simplify**):

(a) (8 pts) $f(x) = \sin^2 x + \sin(x^2) + (x+1)\cos x + \frac{\cos x}{x+1}$.

(b) (8 pts) $f(x) = \tan\left(\frac{1}{x} + \sqrt{1-\sqrt{x}}\right)$

(c) (8 pts) $f(x) = \int_x^{x^2} \sqrt{1+t^3}\,dt + \int (x^{2008}+1)^{88} dx$

#3. (12 pts) Use the *definition* of the derivative as a *limit* to calculate $f'(x)$ for $f(x) = \frac{1}{x}$. (There will be no credit for other methods).

#4. (15 pts)

(a) (10 pts) Let y be the implicit function of x determined by $y^2 - 2x - 4y - 1 = 0$. Find $\frac{dy}{dx}$.

(b) (5 pts) Using part (a), find the equation of the line tangent to the curve $y^2 - 2x - 4y - 1 = 0$ at the point $P(-2, 1)$.

#5. (12 pts) When a circular plate of metal is heated in an oven, its radius increased at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 10 cm?

#6. (12 pts) A rock thrown vertically upward reaches a height of $s = 24t - 0.8t^2$ meters in t seconds. How long does it take the rock to reach the highest point? And how high does the rock go?

#7. (32 pts) Evaluate the following integrals:

(a) (8 pts) $\int_1^4 \left(2x - \frac{1}{\sqrt{x}}\right) dx$.

(b) (8 pts) $\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}\cos\sqrt{x}}{2\sqrt{x}} dx$

(c) (8 pts) $\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|) dx$

(d) (8 pts) $\int t^{-2} \sin\left(1 + \frac{1}{t}\right) dt$

#8. (10 pts) Find the linearization of the function $f(x) = \sqrt{x}$ at the point $a = 25$. Then, using this linearization, estimate $\sqrt{25.2}$. (No credit will be given for any other methods.)

#9. (10 pts) Consider $f(x) = x^2$ for $x \in [0, 4]$. Sketch the rectangles associated with the upper sum of $f(x)$ over $[0, 4]$ by dividing the interval into four sub-intervals with equal length, and find this upper sum.

#10. (24 pts) Let $y = f(x) = \frac{x^2+3x-3}{x-1}$.

(a) (2 pts) Given that we can write the function as $y = f(x) = x + 4 + \frac{1}{x-1}$. Indicate the vertical asymptote of $y = f(x)$, and the oblique asymptote of $y = f(x)$ as well. (**No** technical details are needed to justify your answer.)

(b) (6 pts) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing. Identify the points where $f(x)$ has a local maximum and the points where $f(x)$ has a local minimum. (It is helpful to use the form $y = f(x) = x + 4 + \frac{1}{x-1}$ to evaluate $f'(x)$.)

(c) (10 pts) Find the intervals on which $f(x)$ is concave-up and the intervals on which $f(x)$ is concave-down.

(d) (6 pts) Sketch the graph $y = f(x)$ using the information from parts (a)-(c).

#11. (15 pts) You are designing a rectangular poster to contain 50 in.^2 of printing with a 4-in. margin at the top and bottom and a 2-in. margin on each side. What overall dimension will minimize the amount of paper used. **Give an argument to show that your answer does give a minimum value.**

#12. (10 pts) Find the area of the region enclosed by curve $y = x^4$ and the line $y = 8x$.