Name: ______________________________

Section: ____________ **Recitation Instructor**: ____________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. (4 points) Find the most general antiderivative of $f(x) = 4\cos x + 8$.

Solution:

$$F(x) = 4\sin x + 8x + C$$

2. (5 points) Evaluate: $\displaystyle\int_1^2 \frac{3-2x^6}{x^4}\,dx$

Solution:

$$\begin{aligned}\int_1^2 \frac{3-2x^6}{x^4}\,dx &= \int_1^2 3x^{-4} - 2x^2\,dx \\ &= \left[-x^{-3} - \frac{2}{3}x^3\right]_1^2 \\ &= \left[-\frac{1}{8} - \frac{16}{3}\right] - \left[-1 - \frac{2}{3}\right] \\ &= 1 - \frac{1}{8} - \frac{14}{3}\end{aligned}$$

3. (5 points) Solve the initial value problem: $\dfrac{dy}{dx} = \dfrac{1}{x^2}, \qquad y(1) = 0$

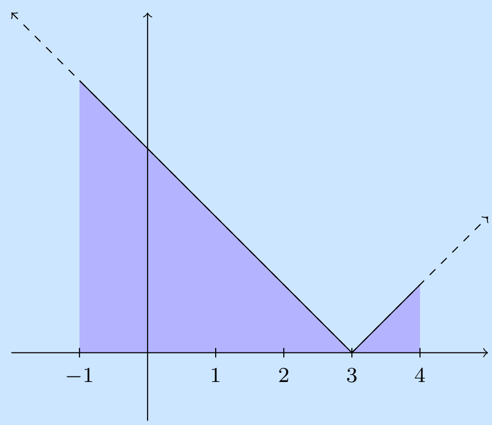
Solution:

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{x^2} \\ y &= -\frac{1}{x} + C \\ 0 &= -\frac{1}{1} + C \\ 1 &= C \\ y &= -\frac{1}{x} + 1\end{aligned}$$

4. (7 points) Roughly sketch the graph of $y = |3 - x|$ and use your sketch to evaluate $\int_{-1}^{4} |3 - x|\, dx$

Solution: Using area formulas for triangles we see that

$$\int_{-1}^{4} |3 - x|\ dx = \tfrac{1}{2}(4)(4) + \tfrac{1}{2}(1)(1)$$
$$= 8 + \tfrac{1}{2} = 8.5 = \frac{17}{2}$$



5. (7 points) Find all critical numbers (i.e., critical points) of the function $f(x) = x^2(9 - x)^{\frac{2}{3}}$

Solution:

$$f(x) = x^2(9 - x)^{\frac{2}{3}}$$
$$f'(x) = 2x(9 - x)^{\frac{2}{3}} + x^2[\tfrac{2}{3}(9 - x)^{\frac{-1}{3}}(-1)]$$
$$f'(x) = 2x(9 - x)^{\frac{2}{3}} - \frac{2x^2}{3(9 - x)^{\frac{1}{3}}}$$

and from here we see that the derivative is undefined when $x = 9$.

$$0 = 2x(9 - x)^{\frac{2}{3}} - \frac{2x^2}{3(9 - x)^{\frac{1}{3}}}$$
$$2x^2 = 6x(9 - x)^1$$
$$2x^2 = 54x - 6x^2$$
$$8x^2 - 54x = 0$$
$$2x(4x - 27) = 0$$

and since $x = 0, x = 27/4$, and $x = 9$ are all in the domain of f they are all critical numbers.

6. (7 points) Let $F(x) = \int_{x^3}^{4} \frac{1}{t^2+2}\, dt$. Find $F'(x)$ ***without actually finding*** $\mathbf{F}(\mathbf{x})$.

Solution:

$$F(x) = -\int_{4}^{x^3} \frac{1}{t^2+2}\, dt$$
$$F'(x) = -\frac{1}{(x^3)^2+2} \cdot 3x^2$$
$$F'(x) = \frac{-3x^2}{x^6+2}$$

7. (7 points) Find the absolute maximum and absolute minimum values of

$$f(x) = 4x^3 - 9x^2$$

on the interval $[-1, 2]$.

Solution: $f'(x) = 12x^2 - 18x = 6x(2x-3)$ which has critical points when $x = 0$ and when $x = 3/2$ so

$$f(-1) = 4(-1) - 9 = -13$$
$$f(0) = 0$$
$$f(3/2) = \tfrac{27}{2} - \tfrac{81}{4} = \tfrac{-27}{4} = -6.75$$
$$f(2) = 32 - 36 = -4$$

so the absolute maximum is 0 and the absolute minimum is -13.

8. Consider $f(x) = x^3 - 3x^2$.

(a) (2 points) Find the x-intercept(s) of $y = f(x)$.

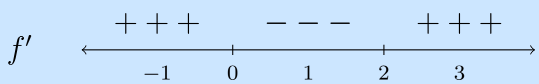
Solution:

$$0 = x^3 - 3x^2$$
$$0 = x^2(x-3)$$

so there are x-intercepts at $x = 0$ and $x = 3$.

(b) (4 points) Find the interval(s) where f is increasing and where f is decreasing. Express your answers using interval notation.

Solution: $f'(x) = 3x^2 - 6x = 3x(x-2)$ which is never undefined but is 0 when $x = 0$ and when $x = 2$.
Using a number line and choosing test points we see that

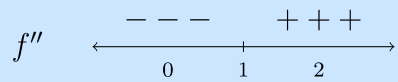


and so

f is increasing on $(-\infty, 0) \cup (2, \infty)$
f is decreasing on $(0, 2)$

(c) (4 points) Find the interval(s) where f is concave up and where f is concave down. Express your answers using interval notation.

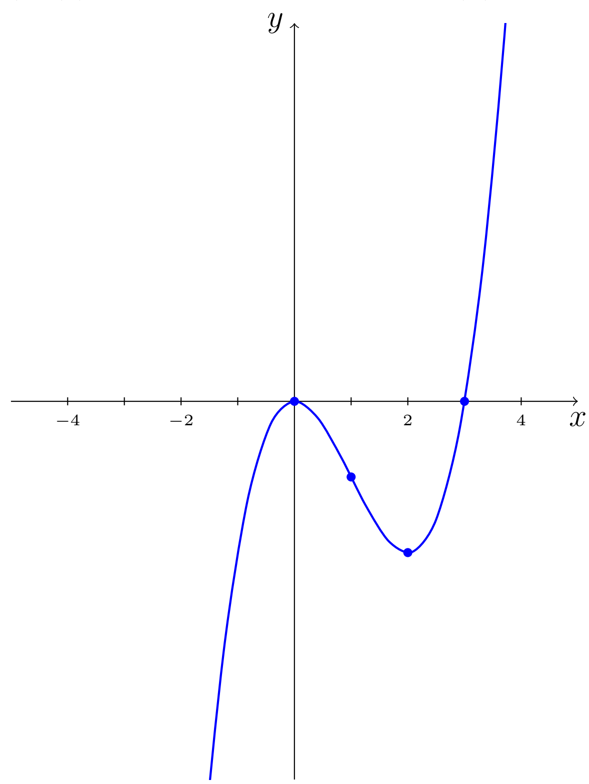
Solution: $f''(x) = 6x - 6 = 6(x-1)$ which is never undefined but is 0 when $x = 1$.
Using a number line and choosing test points we see that



and so

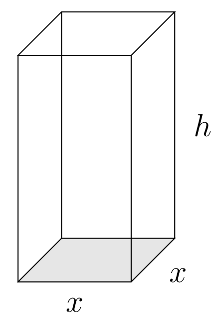
f is concave up on $(1, \infty)$
f is concave down on $(-\infty, 1)$

(d) (4 points) Using the information in parts (a)−(c), sketch the curve of $y = f(x)$.



9. (14 points) A box with square base and open top is to have a volume of 32 in^3. Find the dimensions of the box that minimizes the amount of material used.

Use techniques of calculus to justify that your answer is a minimum.



Solution: The restriction equation is

$$32 = x^2 h$$
$$\frac{32}{x^2} = h$$

We are trying to minimize

$$A = 4(xh) + x^2$$
$$A(x) = 4x\left(\frac{32}{x^2}\right) + x^2$$
$$A(x) = \frac{128}{x} + x^2 \qquad (x > 0)$$

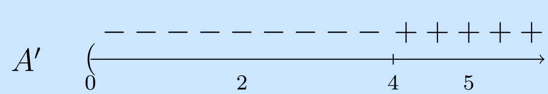
now we find critical points

$$A'(x) = \frac{-128}{x^2} + 2x$$

never undefined when $x > 0$ so setting the derivative to 0...

$$0 = \frac{-128}{x^2} + 2x$$
$$\frac{128}{x^2} = 2x$$
$$64 = x^3$$
$$4 = x$$

Now we verify that $x = 4$ minimizes the amount of material used.



$$A'(2) = -32 + 4 < 0 \text{ and } A'(5) = -128/25 + 10 > 0$$

and so $\boxed{x = 4 \text{ in. and } h = \frac{32}{16} = 2 \text{ in.}}$ minimizes the amount of material used.

Multiple Choice. Circle the single best answer. No work needed. No partial credit available.

10. (4 points) Use a left sum with 4 equally-spaced rectangles to approximate $\int_{-2}^{6} (x^2+1)\, dx$.

A. 38

B. 56

C. 248/3

D. 120

E. 128

11. (4 points) One of the following is an odd function, Which one?

A. $y = x + \sin x$

B. $y = x + \cos x$

C. $y = x^2 + \sin x$

D. $y = x^2 + \cos x$

E. $y = 1$

12. (4 points) Which of the following is the equation of a horizontal asymptote for the curve $y = \dfrac{9x-2}{5-2x}$?

A. $y = \dfrac{9}{2}$

B. $y = -\dfrac{9}{2}$

C. $y = 0$

D. $y = -\dfrac{5}{2}$

E. $y = \dfrac{5}{2}$

13. (4 points) Use linear approximation to approximate $\sqrt{35}$.

A. $6 - \frac{1}{12}$

B. $6 + \frac{1}{6}$

C. $6 + \frac{1}{18}$

D. $6 - \frac{1}{6}$

E. $6 + \frac{1}{9}$

14. (4 points) The graph of the first derivative $f'(x)$ of a function $f(x)$ is shown. At what value of x does f have a local maximum?

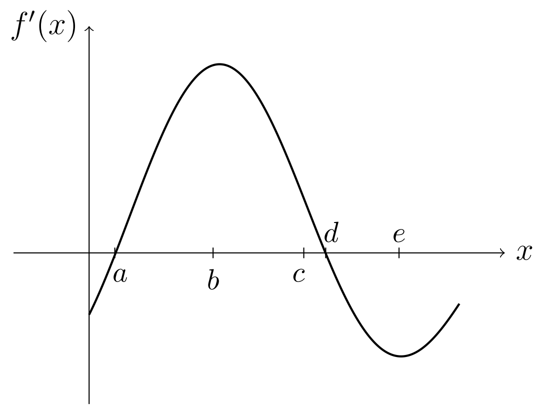
A. $x = a$

B. $x = b$

C. $x = c$

D. $x = d$

E. $x = e$



15. (4 points) Evaluate the sum $\sum_{i=1}^{30}(3+2i)$

A. 1020

B. 930

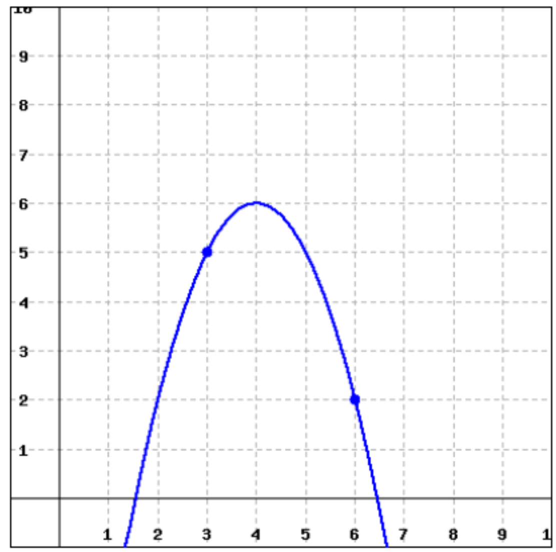
C. 990

D. 555

E. 63

16. (4 points) The function $f(x)$ is given by the graph below. Find the best value c that satisfies the conclusion of the Mean Value Theorem for $f(x)$ on $[3, 6]$.

Hint: *Use the Mean Value Theorem stated on the formula sheet.*



A. $c = 3$

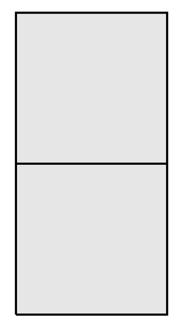
B. $c = 4$

C. $c = -1$

D. $c = 6$

E. $c = 4.5$

17. (4 points) A farmer has 60 meters of fence, and wants to build a rectangular pig pen with a fence in the middle, as shown. What is the area of the largest possible pig pen?



A. 150 m^2

B. 400 m^2

C. 60 m^2

D. 360 m^2

E. 120 m^2

18. (4 points) Use Newton's Method to approximate a solution to the equation $x^5 + x = 35$ starting with $x_1 = 2$. Then $x_2 =$?

A. $x_2 = -79$

B. $x_2 = 83$

C. $x_2 = 81$

D. $x_2 = 161/81$

E. $x_2 = 163/81$

Congratulations you are now done with the exam!

Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	14	
7	12	
8	12	
9	12	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1,y_1) to (x_2,y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
  $= 1 - 2\sin^2\theta$
  $= 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE

Limits

- $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(fg)' = f'g + fg'$
- $\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$
- $(f(g(x)))' = f'(g(x)) \cdot g'(x)$
- Trig derivatives

$$\begin{aligned} &(\sin x)' = \cos x, && (\cos x)' = -\sin x \\ &(\tan x)' = \sec^2 x, && (\cot x)' = -\csc^2 x \\ &(\sec x)' = \sec x \cdot \tan x, && (\csc x)' = -\csc x \cdot \cot x \end{aligned}$$

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.
- (MVT) If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ that satisfies $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.
- (FToC P1) If $F(x) = \int_a^x f(t)\ dt$ then $F'(x) = f(x)$.

Other Formulas

- Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
- Linearization of f at a: $L(x) = f(a) + f'(a)(x-a)$
- $\sum_{i=1}^{n} c = cn$
- $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$