

1. Approximate $\sqrt{50}$

$$f(x) = \sqrt{x} \quad , \quad f(49) = 7 \quad , \quad f'(x) = \frac{1}{2\sqrt{x}} \quad , \quad f'(49) = \frac{1}{14} .$$

$$\begin{aligned} L_{49}(50) &= f(49) + f'(49)(50-49) \\ &= 7 + \frac{1}{14} \approx \sqrt{50} \end{aligned}$$

2. Approximate the value of $f(x) = x\sqrt{50-x}$ at $x=2$ using the linearization at $x=1$.

$$f'(x) = \sqrt{50-x} + \frac{-x}{2\sqrt{50-x}} = \frac{2(50-x)-x}{2\sqrt{50-x}} = \frac{100-3x}{2\sqrt{50-x}} .$$

$$f'(1) = \frac{100-3}{2\sqrt{50-1}} = \frac{97}{14} \quad , \quad f(1) = 1 \cdot \sqrt{50-1} = 7 .$$

$$L_1(2) = f(1) + f'(1)(2-1) = 7 + \frac{97}{14} .$$

3. Let $f(x) = x + \sin x$, on $[0, 3\pi]$. Find the local and absolute maxima and minima.

$$f'(x) = 1 + \cos x \rightarrow f' = 0 \Rightarrow \cos x = -1 \Rightarrow x = \pi, 3\pi .$$

$$f(0) = 0 \quad , \quad f(\pi) = \pi \quad , \quad f(3\pi) = 3\pi .$$

$\therefore (\pi, f(\pi))$ is not local max or min.

$(0, f(0))$ absolute min

$(3\pi, f(3\pi))$ absolute max.