Math 132 Fall 2017
Quiz 3 9/22/17 Name (Print): SOLUTIONS.

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1. Let $f(x) = 1 - x^2$ and $g(x) = x^3 - x$. Show there exists $c \in (-0, \infty)$ so that $f(c) = g(c)$.

$F = f - g$; $F = 1 - x^2 - x^3 + x$; $F(0) = 1$; $F(2) = 1 - 4 - 8 + 2 =$.

$\therefore \exists\, c \in (0, 2)$ so that $F(c) = 0 \Rightarrow f(c) = g(c)$.

2. Let f be defined as

$$f(x) = \begin{cases} 4 - a(x-2)^2, & x \leq 3 \\ ax^2, & x > 3 \end{cases}$$

then find a such that f is continuous.

$$4 - a(3-2)^2 = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = a \cdot 3^2$$

$$\Rightarrow 4 = a10 \rightarrow a = \frac{2}{5}.$$

3. Find the line tangent to $f(x) = \frac{1}{\sqrt{x}}$ at $x = 2$

$f' = -\frac{1}{2} x^{-\frac{3}{2}}$; $\left(\dfrac{y - 2^{-1/2}}{x - 2}\right) = -\dfrac{1}{2^{5/2}}$.

$f'(2) = -\dfrac{1}{2^{5/2}}$

$f(2) = 2^{-1/2}$

4. Using the limit definition of derivative, find $f'(x)$ for $f(x) = \sqrt{1+x}$

$$\frac{\sqrt{1+x+h} - \sqrt{1+x}}{h} \frac{(\sqrt{1+x+h} + \sqrt{1+x})}{(\sqrt{1+x+h} + \sqrt{1+x})} = \frac{(1+x+h) - (1+x)}{h(\sqrt{1+x+h} + \sqrt{1+x})} = \frac{1}{\sqrt{1+x+h} + \sqrt{1+x}}$$

$$\sim f'(x) = \lim_{h \to 0} \frac{\sqrt{1+x+h} - \sqrt{1+x}}{h} = \lim_{h \to 0} \frac{1}{\sqrt{1+x+h} + \sqrt{1+x}} = \frac{1}{2\sqrt{1+x}}$$