

For problems 1 - 3, for each function $f(x)$ and value $x = a$ find $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$, and $\lim_{x \rightarrow a} f(x)$ if it exists.

1. $f(x) = \frac{(x+2)^2}{(x-1)^3 x}$ and $x = 3$. eval @ $x=3 \Rightarrow$ case: $\frac{N}{N}$

\therefore substitute to evaluate:

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3} f(x) = f(3) = \frac{(3+2)^2}{(3-1)^3 \cdot 3} = \frac{25}{24}$$

2. $f(x) = \frac{(x+1)^2}{(x-1)^2}$ and $x = 1$. case $\frac{N}{0}$ notice: denominator ≥ 0 . + $(x-1)^2 = 0$ @ $x=1$.
 $\therefore \frac{N}{0}$ evaluates to ∞ .

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x) = \infty$$

3. $f(x) = \begin{cases} (x+1)^2, & x < 1 \\ 0, & x = 1 \\ 4x, & x > 1. \end{cases}$ and $x = 1$.

left + right limits:

$$\lim_{x \rightarrow 1^-} f(x) = 4$$

$$\lim_{x \rightarrow 1^+} f(x) = 4$$

left limit = right limit +

\therefore limit exists.

$$\lim_{x \rightarrow 1} f(x) = 4.$$

4. Finally, consider $f(x) = 2x - 3$ and note $\lim_{x \rightarrow 2} f(x) = 1$. For any $\epsilon > 0$ find $\delta > 0$ so that $|x - 2| < \delta$ implies $|f(x) - 1| < \epsilon$.

Say $x = 2 + h$

$$|2(2+h) - 3 - 1| < \epsilon$$

$$|4 + 2h - 4| < \epsilon$$

$$|2h| < \epsilon$$

$$|2h| < \epsilon$$

$$|h| < \epsilon/2$$

$$h = x - 2$$

$$|x - 2| < \epsilon/2 = \delta$$