Math 132 Quiz 12 | 12/8/17 | Fall 2017 Name (Print): Solutions

1. Find the following:

a. $\int \frac{1}{\sqrt{x}} \sec^2(1+\sqrt{x})dx$ $= \int \sec^2(u)\,du = \tan(u) + C = \tan(1+\sqrt{x}) + C$

$u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}$

b. $\int_0^1 x^2\sqrt{1+x}dx$ $= \int_1^2 (u-1)^2\sqrt{u}\,du = \int_1^2 \left(u^2\sqrt{u} - 2u\sqrt{u} + \sqrt{u}\right)du$

$u = 1+x$

$du = dx$

$x^2 = (u-1)^2$

$x = \cancel{0} \to u = 1$

$x = 1 \to u = 2$

$= \frac{2}{7}u^{\frac{7}{2}} - 2\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\Big|_1^2$

$= \left(\frac{2}{7}2^{\frac{7}{2}} - \frac{4}{5}2^{\frac{5}{2}} + \frac{2}{3}2^{\frac{3}{2}}\right) - \left(\frac{2}{7} - \frac{4}{5} + \frac{2}{3}\right)$

2. Find the area between the curves $f(x) = 3 - 2x^2$ and $g(x) = x^2 + x - 1$ on $[0, 2]$

$f(x) = g(x) \leftrightarrow 3 - 2x^2 = x^2 + x - 1$

$\leftrightarrow 0 = 3x^2 + x - 4$

$x_0 = \frac{-1 \pm \sqrt{1+48}}{6} = 1, -\frac{4}{3}$

$1 \in (0,2)$; $-\frac{4}{3} \notin (0,2)$.

$f > g$ on $(0,1)$

$f < g$ on $(1,2)$

$\therefore$

$\text{Area} = \int_0^1 \{(3-2x^2) - (x^2+x-1)\}dx + \int_1^2 \{(x^2+x-1) - (3-2x^2)\}dx$

$= \int_0^1 (4 - x - 3x^2)dx + \int_1^2 (3x^2+x-4)dx = \left(4x - \frac{x^2}{2} - x^3\right)\Big|_0^1 + \left(x^3 + \frac{1}{2}x^2 - 4x\right)\Big|_0^2$