Math 132 Fall 2017
Quiz 11 12/1/17 Name (Print): Solutions.

1. Suppose a tank is leaking water. At time 0 the tank has 50 liters of water an it leaks water at a rate of $r(t) = 50(1+t)^{-2}$ liters per minute. How long does it take until the tank is half empty?

TWO METHODS:

(i) Total leaked:

$$\int_0^t r(s)ds = \int_0^t 50(1+s)^{-2}ds$$

$$= -50 \frac{1}{1+s}\Big|_0^t$$

$$= 50 - 50\frac{1}{1+t}$$

$$= 50\frac{t}{1+t} = \frac{50}{2}$$

$$\frac{t}{1+t} = \frac{1}{2}$$

$$\therefore \boxed{t=1}$$

(ii) Initial value problem

$V \equiv$ amount in tank.

$$V'(t) = -r(t) = -50(1+t)^{-2}$$

$$V(t) = 50\frac{1}{1+t} + C$$

$$V(0) = 50\frac{1}{1+0} + C = 50 \to C = 0$$

$$V(t) = 50\frac{1}{1+t} = 25 = \frac{50}{2}.$$

$$\therefore \boxed{t = 1.}$$

2. Let $G(x) = \int_0^x (5+u^2)du$ on $0 < x < 5$. Apply the mean value theorem to G on the interval $[0,5]$ and find all c satisfying the mean value theorem.

$$G(x) = \int_0^x (5+u^2)du = \left(5u + \tfrac{1}{3}u^3\right)\Big|_0^x = 5x + \tfrac{1}{3}x^3$$

$$\frac{G(5) - G(0)}{5-0} = 5 + \frac{25}{3} = \frac{40}{3} = G'(c)$$

$$G'(x) = 5 + x^2$$

$$5 + \frac{25}{3} = 5 + c^2$$

$$c = \pm\frac{5}{\sqrt{3}} \quad \text{but } c \in (0,5)$$

$$\therefore \boxed{c = \frac{5}{\sqrt{3}}}$$